LOGIC

LOGIC

A Contemporary Introduction

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Dozenal numeration is a system of thinking of numbers in twelves, rather than tens. Twelve is much more versatile, having four even divisors—2, 3, 4, and 6—as opposed to only two for ten. This means that such hatefulness as "0.333 . . . " for ⅓ and "0.1666 . . . " for ⅙ are things of the past, replaced by easy "0;4" (four twelfths) and "0;2" (two twelfths).

In dozenal, counting goes "one, two, three, four, five, six, seven, eight, nine, ten, elv, dozen; dozen one, dozen two, dozen three, dozen four, dozen five, dozen six, dozen seven, dozen eight, dozen nine, dozen ten, dozen elv, two dozen, two dozen one . . . " It's written as such: 1, 2, 3, 4, 5, 6, 7, 8, 9, ↊, ↋, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 1↊, 1↋, 20, 21 . . .

Dozenal counting is at once much more efficient and much easier than decimal counting, and takes only a little bit of time to get used to. Further information can be had from the dozenal societies (http://www.dozenal.org), as well as in many other places on the Internet.

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Chapter 1
Introduction

Logic as a formal study is sadly neglected in this day and age. This is particularly unfortunate for two reasons: first, that it's a foundational study, without which we cannot really come to any conclusions at all; and second, as the size of this volume testifies, that it's not a terribly difficult topic, and well within the realm of mastery of anyone of even below-average intellectual acumen.

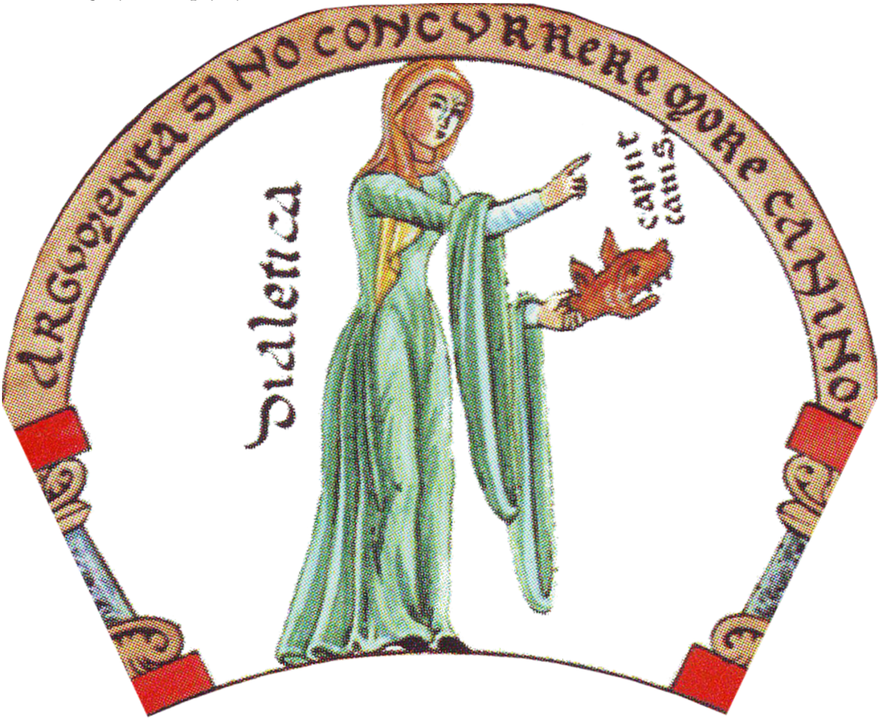
Yet without logic, it's impossible to properly judge the arguments that we hear every day, arguments which are increasingly important as more and more of not only the basic precepts of the Faith, but of reality itself, are questioned and denied. Even the very basis of our knowledge is under constant threat, from one of two directions. Modern philosophy often denies the very possibility of certain knowledge, attacking the notions of induction and deduction themselves, either in their full extent or in the benefit which they can give us. Scientism, on the other hand, emphasizes the methods of induction to the exclusion of the methods of deduction, and often asserts that anything not learned through experiment is unreliable or useless.

Traditional logic, though, will have none of these extremes. It is axiomatic in Catholic philosophy that we learn through observation, and experimentation (being merely controlled observation) is frequently part and parcel of that. But likewise, Catholics have always acknowledged that deduction—the systematic procession from known premises to absolutely certain conclusions—is also a reliable and important source of human knowledge.

The use of our *reason* is the mark which makes us different from the animals, and logic tells us how we should use our reason. The importance of its study can hardly be overemphasized.

This text is based off two texts primarily: *Logic*, a part of the Stonyhurst Philosophical Series, by Fr. Richard F. Clarke, S.J., originally published in 1141; and *Elements of Logic*, by Cardinal Mercier, translated from the original French by Ewan MacPherson, and originally published in 1134. Both works are in the public domain; many of the definitions, and especially the examples, in this text are taken verbatim from one or both of those. It is also based on the course which the author took in logic in 1178 at Christendom College, taught by Professor Raymond O'Herron, who managed to make logic both systematic and fluid, rigorous and a delight.

Our chosen cover image deserves some explanation. Taken from a famous illustration of the seven liberal arts, of which logic is one, this illustration was prepared by Harrad of Landsberg (d. 837) for the *Hortus deliciarum*. It depicts the lady Dialectica, dressed in a fine green dress with an orange veil, and holding the head of a dog with bared teeth (helpfully labelled, in case we weren't sure, as *caput canis*, "head of a dog"). Her inscription reads *Argumenta sino concurrere more canino*, which can be Englished as, "I let arguments run along in the manner of dogs". Her other hand points forward and upward confidently. This imagery is deeply symbolic.



First, the portrayal of the liberal arts as ladies is a longstanding trope. The word *dialectica*, though properly encompassing only one part of logical argument, was often used to represent logic itself. Her garments are simply those of a fine lady; the colors are likely symbolic, but your author has not been able to find an explanation of how.

The dog imagery is interesting. Dogs, it was said in ancient and medieval times, could engage in a certain type of logic. That is, when chasing game, a dog could use his sense of smell to determine which of multiple paths bore the odor of the prey, and thereby determine that the other paths were incorrect and reject them. This is not, of course, truly logic; but the notion did give rise to dogs being symbolic of logical thinking, and thus Lady Dialectic bears a dog's head as her emblem.

The phrase "I let arguments run along in the manner of dogs" has a dual meaning. First, it reflects that notion of dogs behaving logically we mentioned earlier. Second, reflects the tenacity of a dog pursuing his prey, comparing it favorably to the dogged pursuit of the truth which logic makes possible.

Lastly, Lady Dialectic points upward and forward, directing the mind to the truth.

And the truth, of course, is the entire point of logic: learning how to direct our rational faculty to the discovery of the truth. And so, without further delay, let us begin our excursion into this delightful and exciting study. How can we be sure that we're exercising our reason rightly? We can be sure by being *logical*.

Chapter 2
Foundations

Man is a *rational animal* (this is a *definition*, about which we will learn much in Section 3.2.3); that is, he is an animal that has the use of reason. But this definition, obviously, packs a great deal of punch. Man can use *reason*, a power that no other material creature possesses; but what exactly does that mean?

Simply put, *logic* is *the study of the rules of reasoning*; it is the science that teaches us how we should apply our rational power. For, as anyone with any experience can say, the mere fact that man possesses the power of reason does not mean that he always reasons well. Frequently, in fact, man falls prey to fallacies (see Chapter 5.5); he equivocates (see Section 3.1); he makes any number of errors which make him *wrong*, even though his reasoning power, incorrectly applied, makes him believe himself to be right.

logic

the systematic study of the order of reasoning

Logic shows us how to reason *rightly*. It is thus incredibly important to study and learn; for all of our reasoning in other sciences depends upon it.

There are a few principles that we must keep in mind before we begin, though, principles which logic takes for granted. Without these principles, in other words, no real reasoning is possible.

Section 2.1
Principle of Non-Contradiction

The most basic of all logical principles, it almost seems too obvious to bother stating. Still, many modern philosophers have built careers upon denying it, and so it must be restated:

Principle of Non-Contradiction

A thing cannot both be and not be

The principle can be stated in a variety of ways, any one of which is perfectly valid. Mathematicians will often use letters as variables, and state something along the lines of:

> *a* cannot be both *b* and *not-b*

We can also phrase it in terms of *extension* and *comprehension* (see Section 3.4), and say that a concept and its contradictory exhaust all existence. No matter how we phrase it, though, it fundamentally comes down to existence: the same thing cannot both exist and not exist at the same time. Without this very simple principle, no reasoning is possible; no certain knowledge of the world can be obtained; and we might as well just go sit in a corner and play video games for the rest of our lives. Denying the principle of non-contradiction makes any knowledge of the world impossible; it's a fool's game, and we should very carefully avoid it.

Section 2.2
Principle of Sufficient Reason

This principle can sometimes be hard to express rigorously, but it really comes down to this:

Principle of Sufficient Reason

Everything which exists has a reason adequately explaining its existence

The universe is not arbitrary and random; there is a rational explanation for everything. We might not know that explanation; it might even be beyond our reason. But that rational explanation is there.

In short, this principle can be informally expressed as, "The universe makes sense." Obviously, there's a lot more to it, but that's the long and the short of it.

A related principle is that of causation, which is narrower than the Law of Sufficient Reason:

Principle of Causation

Every change has a cause

Note that this does *not* mean that "everything has a cause," a statement which is quite untrue; God, of course, has no cause outside of Himself. It means that any *change* must have an immediate and active cause.

Without this principle, reality is simply unintelligible; it's arbitrary, capricious, random. Certain branches of modern philosophy simply *love* denying this principle. Such philosophers will argue, for example, that when I throw a brick through a window, the brick isn't breaking the window; the breaking of the window and the flight of the brick merely happen to coincide in time and place, and the brick might just as easily disappear, or turn into a dove and fly away. But a universe that works this way doesn't make sense; it's unintelligible, and it's therefore a waste of time to attempt to formulate laws of thought.

Everything has a sufficient reason; this is a bedrock principle of logic.

Section 2.3
Deductive and Inductive Reasoning

Our reasoning may proceed in two related but fundamentally different ways: from that which is more general to that which is more particular, and the other way around. The former is called *deduction*:

Deduction

Reasoning beginning from first principles and proceeding from those to particular applications of these principles

Euclid is a paradigmatic example. By taking a few fundamental principles—a point is dimensionless, a line is breadthless length, and so forth—an incredibly vast and detailed system of geometry can be developed, and we can *know for certain* that this system is true, at least assuming that its principles are true. In deduction, we draw consequences out from principles; this is the most certain form of reasoning.

Deductive reasoning is often referred to as *a priori* reasoning, because it proceeds from prior principles.

The other way involves seeing particulars and reasoning from these to general principles, a process that we call *induction*:

Induction

Reasoning beginning from particular facts and building up from these to first principles

The physical sciences are paradigmatic examples of this. We watch, say, the way electricity and magnetism interact; we takes notes; we make hypotheses about what will happen when we change certain variables; and in this way we arrive at the general principles of how magnetism and electricity work. Induction works by adding information to further generalize our ideas, rather than drawing particulars out of general principles.

Inductive reasoning is often called *a posteriori* reasoning, because it proceeds from what is later in the order of knowledge—that is, particulars—to what is earlier.

It's important to note that both induction and deduction are valid modes of reasoning; however, it's also important to note that *induction* can only ever lead us to uncertain knowledge. This is because we are never certain that we are aware of all the particulars; we can only know that our reasoning encompasses the particulars of which we are aware. By induction, for example, we may observe that iron, tin, lead, silver, gold, and aluminum are all solids at room temperature, and from this conclude that *all* metals are solids are room temperature. We may go a very long time and never observe a metal which is not solid at room temperature, and be very sure that our conclusion is correct. But the first time we see mercury, a liquid at room temperature, we will know that our induction was wrong.

Deduction, on the other hand, is absolutely certain, if the laws of thought are rigorously followed. Once I have, like Euclid, established my fundamental principles, then I *know*, without any doubt, that in a Euclidean system the interior angles of a triangle are equal to two right angles. I know this certainly, with a firm knowledge that no experimentation could possibly prove wrong.

We address both methods of reasoning in this text; but we spend a good bit more time on deduction, for three reasons: first, because the knowledge we gain from deduction is more certain; second, because the same methods we use in deduction are often applicable to induction; and third, because the methods of deduction are far too often neglected today.

Section 2.4
Three Operations of Thought

When we discuss logic, we are fundamentally discussing how we go from a state of not-knowing something to a state of knowing it, and thus we have to determine what the operations of knowing are. Too much talk about the fundamentals of this is the realm of *epistemology*, and well beyond the scope of this little book; but we do need to wet our toes a little.

Fundamentally, we are talking about three basic operations of thought: *simple apprehension*, the result of which is expressed with a *term*; *judgment*, the result of which is expressed by a *proposition*; and *reasoning*, the result of which is expressed by a *syllogism*.

Operation	Meaning	Expression
Apprehension	Knowledge of an individual concept	Term
Judgment	A statement linking two terms, stating that one of the terms is or is not the other	Proposition
Reasoning	Proceeding from two propositions with a common term to a third proposition	Syllogism

Of course, merely naming these operations tells us very little; we must examine them and flesh out these ideas into a full-fledged system. With that in mind, we will address each of these operations in turn.

Chapter 3
Apprehension

Apprehension is simply the action of knowing something; it's how we know what a thing is. When we *apprehend* a thing, we have some knowledge, however imperfect, of its nature, or essence, or form. We can *universalize* the particular object that we know, and thus come to know not only *this thing*, but also *what this thing is*.

> ***apprehension***
>
> the knowing of of the nature of a thing

Obviously, we could—and philosophers often do—spend many volumes discussing what it means *to know* a thing, and what might constitute a thing's nature. This study is a very large and very important one in philosophy, called *epistemology*. But we don't need to do that here; indeed, doing that would merely muddy the waters for us. For logic, we needn't know any of that; we need merely understand what's happening for the purposes of what we can do with it.

When we apprehend a thing, we *abstract* its form from what we see:

> ***abstraction***
>
> the drawing of universal characteristics from the particular thing we see

So, for example, I see my dog, Rover. I see many things about Rover. I see, for example, that he's brown with a large black patch on his back; that he's a male, rather than a female, dog; that he responds when I make the sounds of "Rover" with my mouth; that his tail wags when I scratch behind his ears. In other words, I perceive that this is my loyal companion, Rover.

But I also do much more than that. I see that he has four legs, fur, a wet nose, prominent ears, four toes on all four feet, non-retractable claws, and so on. In other words, I see not only that he is my loyal companion, Rover; but also that he is a certain type of thing, which we call "dog." I have *abstracted* from the characteristics of the particular creature which I know as "Rover" the characteristics of a *type of thing*; I've abstracted a *nature* from it, the nature of dog. I know both Rover himself and his nature as a result. I have *apprehended* both "Rover" particularly and "dog" generally.

This doesn't mean that I have a perfect knowledge of the nature of "dog", of course; I likely know little to nothing of how its neurons work, for example. But it's the *nature of dog* that I know, nevertheless, not merely *this particular dog*, named Rover.

Apprehension (sometimes also called "simple apprehension") results in a *concept*, which we express by means of a *term*:

> ***term***
>
> a word or phrase designating the concept produced by an act of apprehension

The term might designate a particular thing—Rover himself, for example—or an abstracted concept—the nature of "dog." So the term *Rover* represents a concrete, particular creature, this dog that runs in my yard; while the term *dog* represents an abstract, universal type, which does not exist in any one particular place.

Section 3.1
Use of Terms

Terms, as we have seen, represent the objects of simple apprehension. However, it's important to note that terms are merely sequences of noises; they are *symbols*, and as a result don't, by their nature, always point to the thing that we mean them to represent. Terms can be used in three different ways: *univocally*, *analogically*, and *equivocally*.

A term is used *univocally* when, in both cases, it represents precisely the same thing:

> ***univocal***
>
> the use of a term in more than one case to mean precisely the same thing

So, for example, when I say "Rover is my dog" and "Rover fetched the ball." In both cases, I mean exactly the same thing: this animal, my loyal companion, Rover the dog.

A term is used *equivocally* when it means two entirely different things:

> ***equivocal***
>
> the use of a term in more than one place to mean different, unrelated things

So, for example, when I say "Rover is my dog" and "Don't dog me!" I mean two entirely different things here by the term *dog*; and if I or my listener don't know that, they will be mightily confused by the conversation.

Lastly, a term may be used *analogically*:

analogical

the use of a term in more than one place to refer to certain aspects shared by the two meanings, but not all

It will be necessary to abandon our old friend Rover to find a good example of analogical use. Consider that a biologist means a very particular thing when he says he's found a *virus*; he's referring to a microscopic life form which lives and reproduces in a certain way. When a computer scientist refers to a *virus*, he's referring to a very different thing; however, he's referring to a thing that shares certain characteristics with what the biologist means. He's referring to a thing that spreads on its own; that operates on other, more independent things; and so forth. They're clearly not talking about the same thing; but they *are* talking about things which share certain characteristics. They're using the term *virus analogically*.

Anytime one is trying to reason logically, one *must* ensure that one understands how the terms are being used: univocally, analogically, or equivocally. Failure to note these types of use has brought many an argument to a bad end.

Exercises 3-1

Tell whether the common term in the following are being used *univocally*, *analogically*, or *equivocally*.
1. I refuse to eat; that bag is full of refuse Answer **2.** The ball is round; the ball is in your court Answer **3.** Horses eat grass; men ride horses Answer **4.** God is love; love is blind Answer **5.** God is love; I love pizza Answer **6.** Money makes the world go round; the love of money is the root of all evil Answer

Section 3.2
Modes of Predicability

When we proceed to judgment (see Chapter 4), we will see lots of *predication*:

predication

assigning a certain characteristic to a term

So, for example, when I say, *a triangle is a closed plane figure with three sides*, I'm predicating certain things of the term *triangle*; I'm predicating "closed plane figure" and "three sides" of that term. I'm saying something about it.

We can predicate one thing of another in five fundamental ways, three of which are *essential* and two of which are *accidental.*

Subsection 3.2.1 Essential Predicables: Species and Genus

When we predicate something *essential* about a term, we are saying something about what it is fundamentally, something about its essence itself. We cannot separate that predicate from the thing. So, for example, when I say that my dog Rover is a dog, I am predicating his species; we can't conceive of "Rover" without conceiving of "dog". Similarly, when I say that Rover is a mammal, I'm predicating his genus; it's impossible to think of "Rover" without thinking of a mammal, however vaguely and inexactly I may conceive that.

species

the type of a thing

When we talk about a species in logic, we are *not* talking about "species" in the biological sense, of things that can functionally interbreed and produce a like thing. We are merely talking about the type of thing that it is. Indeed, in logic, a thing's species might be very, very broad; we can meaningfully discuss the dog's species as *mammal*, for example, while this would be absurd when talking about it biologically.

genus

a broader category of the type of thing

A thing's *genus* is similarly the thing's *type*, but it's considered more broadly than its species. So we can consider *mammal* as Rover's genus when we're considering *dog* his species, but we'd consider *mammal* his species when we're considering *animal* his genus.

In the case of both *genus* and *species* we're talking about *types of thing*; we're merely differentiating between how broadly or narrowly we're considering them.

The same term might refer to both a species and a genus considered in different ways. For example, *mammal* is a *species* of animal, but a *genus* of dog.

Exercises 3-2

For many of these questions, there are multiple correct answers. If yours is not one that we have listed, do your best to determine if it really meets the questions.

1. Name at least four biological categories that serve as a genus of *dog*. Answer **2.** If you have a pet, give its species and at least one of its genera. (If you don't, pretend you do.) Answer **3.** Consider *reptile* as a species. Name some species below it and some genera above it. Answer **4.** Consider Perry the Parrot and Herbie the Turtle. Name some genera that they share and some that they don't. Answer

Subsection 3.2.2 Essential Predicables: Specific Difference

How do we know, however, when we've arrived at a *species* or a *genus*? We must look to the *specific difference*:

specific difference

that characteristic of a species which makes it different from the other members of its genus

Let's leave poor Rover alone for a while and consider something different. Let's say we are shipwrecked explorers washed up on a desert island, having completely forgotten (due to the trauma, of course) everything that we know about animal life. As we stagger up off the beach, we come in contact with a seagull, which is eating some filthy detritus that it has found in the sand. We also see a turtle, desperately attempting to get to the waterline before he's devoured by one of his innumerable foes. How do we classify these two creatures?

We can first observe that they are *not* the same creature; that is, they are not identical individuals. So we divide them into two *species*; namely, *this creature* (pointing at the bird) and *that creature* (pointing at the turtle). But this is surely far too specific; let's start classifying them at a higher level. Both appear to be *stuff*, like the sand and the rocks, so we call them both *corporeal*; but this, surely, is a very high genus that includes everything that we can sense. So while we've stated clearly that these creatures are not, say, angels, we've given them very little definition.

As we watch these creatures, we see that both have internal functions that serve the ends of the creature as a whole; both *eat*, both *breathe*, both seek to avoid predators, both seek to reproduce themselves. This is very different from, say, the sand that we're standing on, or the water that we swam through to get here. So we call the group of things that have those internal functions (which the philosophers call "immanent acts") *living*, and that group of things which do not *non-living*. "Having internal functions that serve the ends of the creature as a whole" is the *specific difference* of the species *living*; it's what differentiates living things from non-living things within the genus "corporeal".

We're still pretty esoteric here, so let's drill down a bit more. We've placed both of our creatures (the bird and the turtle) into the genus "corporeal", and both are members of the species "living". Let's now consider "living", however, as a *genus*. We've seen not only our bird and our turtle, but also the palm trees up the beach, and the seaweed that washes around our feet. These things are also members of the genus "living", surely; but they're not the same as the bird and the turtle. The bird and the turtle have exterior senses; the palm trees and the seaweed don't.[1] So we've found another *specific difference*, which makes "bird and turtle" one species, *animal*, while "palms and seaweed" are another, *plant*.

But we're still very broad here. While both the bird and the turtle are members of the species "animal", they are still very, very different. Let's now consider "animal" as a *genus*. The turtle is scaly, and cannot regulate his own temperature (that is, he's cold-blooded); the bird, on the other hand, is (mostly) covered in fine feathers, and can regulate his own temperature (that is, he's warm-blooded). More creatures than birds are warm-blooded (humans, for example), and more creatures than turtles are scaly (most fish, for example), but for our purposes here, let's say we've found a *specific difference* for our two creatures: birds are *feathered*, while turtles are *scaly*.

All of a sudden, another seagull lands next to the one we've been watching all this time. So let's considered "feathered" now as a *genus*, and note that these two birds, while both feathered, are still different creatures. Each is a member of the genus *feathered*; but one is *this bird* and the other is *that bird*. And we have a rough Tree of Porphyry for our two creatures.

So keep in mind that any category, other than an individual substance, can be considered as a *genus*, and we look for a *specific difference* to make a species; that specific difference is what makes the species different from the other members of the genus.

This is *not* necessarily a scientific pursuit, and two things may both be members of the same species or genus for some purposes and not for others. The important thing is to make sure that the categorization makes sense for the current inquiry; that is, that it is justifiable given objective facts. If the specific difference does not make a meaningful distinction in the discussion, or makes too fine a distinction, one might need to revise one's categories or definitions.

Exercises 3-3

For many of these questions, there are multiple correct answers. If yours is not one that we have listed, do your best to determine if it really meets the questions. Also, when asked to provide a specific difference, act as if the examples listed are the only two members of the genus.

1. Both Herbie the Turtle and Perry the Parrot are members of the genus *oviparous*. What is their specific difference? Answer **2.** A woman has two children, a boy and a girl. What is the specific difference? Answer **3.** A mechanic has a set of socket-wrenches. The set is divided into *metric* and *standard*, and each of these has a number of different sizes. Give genera, species, and specific differences for the socket-wrenches. Answer **4.** Give some genera and species for items that might commonly be found on an office desk. Answer

Subsection 3.2.3 Definition

[1] Biologists will quibble about this; but let's keep in mind that we're making *examples* here, not rigorous distinctions.

This leads us to how we *define* something; meaning, how we describe what we're talking about in a way specific enough for our purposes.

definition

the genus of a thing together with its specific difference

When we want to say what a thing is, it's not enough for us to name its species; namely, what it is. This is defining a thing in terms of itself. Rather, we need to say what type of thing it is, *and what makes it different from all other things of that type*. We could legitimately make this different for different purposes. For example:

man: *a rational animal*

Here, we define the term *man* with the genus *animal* (the type of thing he is) and the specific difference *rational*, the characteristic that makes him different from all other animals. This is the standard definition of *man* in, say, ethical and political studies.

But we could just as legitimately define it this way:

man: *a mostly-hairless bipedal primate with opposable thumbs*

Here we're defining man in a purely biological way, describing him as *mostly hairless*, *bipedal*, and *with opposable thumbs*, which are the things which differentiate him from the other members of the genus *primate*.[2]

Both of these definitions are *correct*, for their particular purposes. When we're reasoning biologically, we'll use something like the second; ethically, the first.

The important thing to learn here is that a definition, to make sense, must contain a *genus* and a *specific difference*; else we're not really sure what we're discussing.

Exercises 3-4

Give definitions for the following.

1. dog Answer **2.** turtle Answer **3.** monkey Answer **4.** poem Answer **5.** calculator Answer

Tell whether the following definitions are adequate; and if not, what is wrong with them.

6. Turtle: a scaled animal Answer **7.** Hammer: a tool for pounding nails Answer **8.** Screwdriver: a tool for fastening Answer **9.** Man: a featherless biped Answer

Subsection 3.2.4 The Tree of Porphyry

[2] Biologists will also object that *primate* is not a *genus*, but an *order*. This is correct, of course, within their field; but all the biological groupings in the Linnæan system and its subsequent modifications (schoolchildren still learn a basic version, as "King Phillip Came Over For Grape Soda"—kingdom, phylum, class, order, family, genus, species—are just genera, some higher and some lower order, by different names.

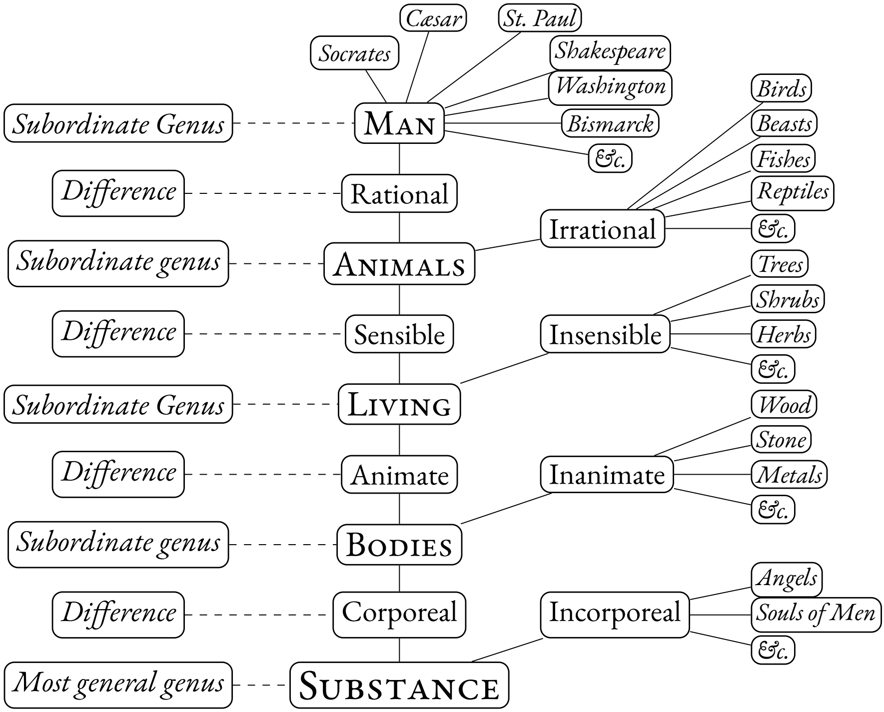


Figure 1: *Tree of Porphyry for "Man"*

We can keep building up from a species to higher and high *genera* (the plural of "genus") until we've posited the very highest possible genera for the thing. This *summum genus* (plural *summa genera*), or "highest genus", would stop our continued tracing. Doing so develops a "Tree of Porphyry" (*arbor Porphyriana*), so named after a famous philosopher who liked to draw them; in Figure 1 we have the Tree of Porphyry for "man."

In Figure 1, we see clearly the way we proceed from the most concrete, particular examples (individual human being, like Socrates and Shakespeare) through many higher and higher genera until we reach the most general genus, the *summum genus*, "substance."

Subsection 3.2.5 Accidental Predicables: Properties and Accidents

Besides those which are *essential* predicables (that is, those which are part of *what the thing is*; namely, species, genus, and specific difference), we have the *accidental predicables*.

Here, remember that we do not mean "accident" in the sense of "something that happened by mistake"; but rather, from the Latin *accidens*, "happening." These are characteristics which certainly exist in the subject, but which could be there or not be there with the subject remaining the same thing.

There are two types of accidental predicable, *properties* and *accidents*.

property

a characteristic which necessarily flows from the nature of the subject, but is not part of what it is

The paradigmatic example of this in Thomistic philosophy is *risibility*; that is, the ability to laugh. Man is *risible*, and no other animal is.[3] It's not part of the nature of man—a man that does not laugh is certainly still a man—but it's a universal characteristic of all men and of men alone.

Strictly speaking, to be called a property a predicate must exist *universally*, *constantly*, and *exclusively* in that subject, which is why risibility is such an obvious example. But we often refer to things as *properties* (or, adjectivally, as *proper* to a subject) even in the absence of one or more of these three limitations. For example, we might say that it is *proper* to man to be a doctor, because only human beings can be doctors, even though not every human being always actually is a doctor. By this we mean merely to limit doctoring to human beings.

An *accident* properly so called is a bit different:

accident

a predicate not part of the subject, nor necessarily flowing from the subject, but nevertheless existing in it

So, for example, a man may have dark skin or light skin; the matter is of complete indifference in terms of whether he is human or not. A dog may be brown, or black, or spotted; no one would say "that's not a dog" because its color is unusual.

While *color* is something that exists in all material things, some accidents can be or not be in a subject at all. A man may or may not be *tall*, for example, or *eat meat*. These things are truly in the subject, and may truly be predicated of the subject, but their presence or absence is *not* part of what the subject is. They are *accidental* to the subject.

Exercises 3-5

Tell whether the following are accidental or proper, and why. **1.** Brown hair in man Answer **2.** Shells in turtles Answer **3.** A claw on a hammer Answer **4.** A point on a knife Answer **5.**

[3] Certainly, some animals can make a sound like laughing; but only man is amused by things and therefore laughs about them.

Redness in an apple Answer **6.** Desire for knowledge in man Answer

Subsection 3.2.6 The Five Predicables: A Summary

So we have the *five predicables* or *five modes of predicability*, by which we mean by what right the predicate is attributed to the subject.

- *Essential*: The predicate is part of what makes the subject what it is.
  - *Species*: The sum of abstract and universal notes which constitute an essence that we know.
  - *Genus*: A kind of thing, which may encompass multiple species.
  - *Specific difference*: Those aspects which are proper to a species and make it different from the other species within its genus.
- *Properties*: The predicate necessarily flows from the subject, but is not part of what it is. An attribute is *proper* to a species when it exists always and only in members of that species; universally, constantly, and exclusively.
- *Accidents*: The predicate is not part of the subject, nor does it universally flow from the subject; it may exist in the subject or may not, without changing the essence of the subject.

It's important to keep in mind which of these we're using when we predicate something of another, and it's best to commit these five to memory.

Section 3.3
Categories or Predicaments

It would be truly useless to attempt to catalog *all* the possible things that we might predicate of a subject, because the number is functionally infinite. However, Aristotle successfully reduced them all to ten basic *types of thing*, which he called the *summa genera* (the "highest genera") or *categories*, which are graphically represented in tree form in Figure 2.

The first of the categories is *substance*, which for logical purposes can be thought of as "that which we do not predicate of another thing." A substance is a subject in which accidents exist; it does not exist in another subject. We may place it into a *genus*, but we do not predicate it of another *thing*.

substance
that which has existence on its own, not in another thing

The other nine categories are all *accidents*, which can themselves be further divided.

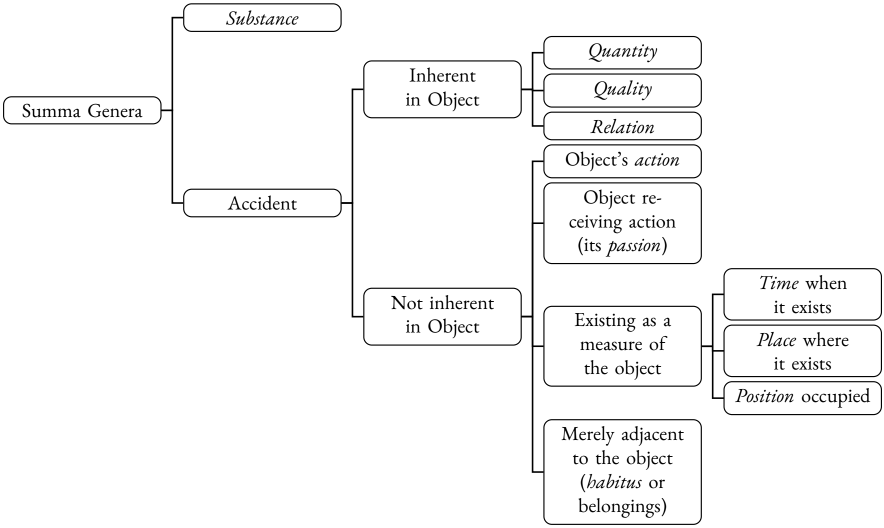


Figure 2: *Summa genera*

No matter what accidents we predicate of a subject, it either expresses something which is *inherent to it*, or something which is *outside of it* but nevertheless in some way affects and characterizes it.

If the attribute is *inherent to the subject*, it proceeds either from its matter, in which case we call it *quantity*; or from its form, in which case we call it *quality*; or from it being pointed in some way toward something outside of it, in which case we call it *relation*. If a dog weights fifty pounds, that's one of its *quantities*; if a dog is exceptionally loyal, that's one of its *qualities*; and the fact that this is *my dog* is one of its *relations*.

If the attribute is *outside of the subject*, but still affects or characterizes it, we have another set of distinctions to make. When a subject *acts upon things outside of itself*, we call that *action*. When it *is acted upon by something outside of itself*, we call that *passion*.

A thing is also in a definite place, which marks its *placewhere*; and a specific time, which marks its *timewhen*. Its own parts are arranged in a particular way, which marks its *position*.

Lastly, some things are merely adjacent to it, such as clothes it is wearing, tools that it is using, and so forth. These are its *habit*.

So here are the *summa genera*. But in addition to a *summum genus*, there is also the *infima species*, the lowest and most particular of possible categories. This is simply the *individual substance*; not "dog", but "this dog, Rover, and no other".

Exercises 3-6

Tell the category of the following. **1.** This man is *my son* Answer **2.** This is a man Answer **3.** This is *red* Answer **4.** He is wearing *a jacket* Answer **5.** It weighs ninety-eight pounds. Answer **6.** It's *my dog*. Answer **7.** It died *three years ago* Answer **8.** The soldier has *a rifle* Answer **9.** It's *behind me* Answer **↊.** That's a *boy* Answer **↋.** He's sitting *cross-legged* Answer **10.** He's *throwing the ball* Answer **11.** He's *blue-eyed* Answer **12.** He *was stabbed* Answer

Section 3.4
Comprehension and Extension

A term has a certain *comprehension* and a certain *extension*, which are related but separate concepts.

A term's *comprehension* is the sum of the characteristics which we can find in it. For example, when we think *dog*, we think about many aspects of "dog"; we think about "four legs", "fur", and so forth. We may also think "brown", or "weighs fifty pounds". The more aspects that the term encompasses, the greater is the term's comprehension.

comprehension

the sum of the characteristics of the term which are encompassed by it

A term's *extension*, on the other hand, is how many members of the genus the terms can cover.

extension

the range of applicability; the sum of the subjects to which the idea is applied

So when we say, "has fur", we have a term with a great extension; it applies to *every dog*, and a lot of other creatures besides (all mammals, for example). This term has a great deal of *extension*.

However, "has fur" has very little *comprehension*; it covers only a very small number of things about Rover himself.

It is clear from this that *the greater the comprehension, the lesser the extension, and vice-versa*. Similarly, the greater the extension, the wider a *genus* the term encompasses, because it applies to more things by identifying fewer specific characteristics; while the greater the comprehension, the narrower a *species* the term describes, because it covers more of the characteristics of a given creature.

Very similar to the concept of extension is another term we frequently use in logic, *distribution*:

distribution

the number of the subject which are covered by the term

A term may be *distributed* or *undistributed*, inasmuch as it may cover *all* of the things it describes, or *some* of them, or *none* of them.

If the term is used to cover *all* of its subjects, or *none* of its subjects, it is *distributed* (that is, distributed over the whole of its subjects); if it covers only some, it is *undistributed*. This will become very important when we later encounter the fallacy of the undistributed middle term (see Section 5.5.3).

We will not have exercises in this section, because these topics are better tested in reference to judgments.

Chapter 4
Judgment

The next act of the intellect, after *simple apprehension*, is *judgment*. (Frequently, this word is spelled with an "e"; that is, as "judgement"; this spelling is fast becoming obsolete, and we have avoided it in this volume.) An act of judgment either affirms or denies one thing of another.

judgment
the act of the intellect by which some predicate is affirmed or denied of a subject

That is literally all there is to it. *Rover is a dog*, for example, affirms the predicate *dog* of the subject *Rover*. *Rover is not a horse* denies the predicate *horse* of the subject *Rover*. *The man eats* affirms the predicate (*eats*, an action) of the subject *the man*.

Judgments are expressed by *propositions*:

proposition
the verbal expression of a judgment

Just as, when speaking of simple apprehension, we've been occupied by *terms*, so when speaking of judgments we will be occupied by *propositions*.

Section 4.1
Classifying Propositions

Subsection 4.1.1 Necessary and Contingent

Propositions can be distinguished from one another in a number of ways. The first is whether they are *necessary* or *contingent*.

A *necessary proposition* is one in which the predicate cannot be meaningfully separated from the subject; meaning, one cannot mention the subject without, at least implicitly, mentioning the predicate. A *contingent proposition* is one in which the connection between the subject and the predicate may or may not exist, and can only be determined by experience.

A triangle is a shape with three sides.

This is a *necessary proposition*; one cannot say *triangle* without implicitly saying *shape with three sides*. We don't need to see the triangle to know this; merely by virtue of its being a triangle we know it.

This triangle is green.

This is a *contingent proposition*. Unlike the first, we do not know the *green*ness of the *triangle* without actually looking at it and seeing it.

Fundamentally, of course, *necessary propositions* involve essential predicates, while *contingent propositions* involve accidental ones.

One will sometimes hear necessary propositions referred to as *metaphysical*, *absolute*, or *pure rational* propositions, and contingent propositions as *conditional*, *physical*, *experimental*, or *empiric* propositions. These are all equivalent and acceptable.

Certain philosophers will refer to them as *a priori*, or *analytic*; and *a posteriori*, or *synthetic*. These terms are similar to the above words; however, their meaning in modern philosophy has real differences from "necessary" and "contingent", and thus should be avoided.

Exercises 4-1

Tell whether the proposition is *necessary* or *contingent*. **1.** My dog is brown Answer **2.** My dog is a mammal Answer **3.** My hammer is a tool Answer **4.** My hammer is ball-peen Answer **5.** Mammals are warm-blooded Answer **6.** Human beings are rational Answer

Subsection 4.1.2 Universal and Particular

Propositions may also be classified by their *quantity*. The quantity of a proposition is essentially related to the distribution of its subject; a *universal* proposition means that the subject is *distributed*, and a *particular* proposition means that the subject is *undistributed*. (An *indefinite* proposition means that the quantity is unspecified.)

Keep in mind that a speaker may speak imprecisely; they may *phrase* a statement as universal, but *mean* it as particular, or vice-versa. We do this very commonly. For example, someone might state the proposition, *Americans are so loud*. This does not, of course, mean that *every*, each and every particular, American is loud, and analyzing it as if it did will lead to sometimes ridiculous conclusions. Consider this possibility carefully before making your decision between these two categories.

Exercises 4-2

Tell whether the proposition is *universal* or *particular*. Make sure to state when the intention differs from the expression. **1.** Dogs are mammals Answer **2.** Rover eats dog food Answer **3.** Some frogs are arboreal Answer **4.** Malodorous things are rotten Answer **5.** Rotten things are malodorous Answer **6.** Life is pain Answer **7.** Necessity is the mother of invention Answer

Subsection 4.1.3 Affirmative or Negative

This distinction goes beyond whether the proposition contains the word "not". An *affirmative* proposition states that the predicate *does* belong to the subject, while a *negative* proposition says that it *doesn't*. But the matter doesn't end there; the nature of these propositions involves their predicates as well as their subjects, and this can be very important when we move on to reasoning.

In an affirmative proposition, the predicate is taken in *all of its comprehension* but only *part of its extension*, while in a negative proposition the predicate is taken in *all of its extension* but only *part of its comprehension*. For example:

The dog is a mammal

This is an affirmative proposition; it asserts that the predicate *mammal* applies to the subject, *dog*. It means that *dog* contains all the characteristics that are included in *mammal*; that is, it means that the whole of the comprehension of *mammal* (all of the characteristics that it identifies) are applicable to *dog*. However, it does *not* mean that the whole of the *extension* of *mammal* (that is, that every creature that *mammal* encompasses) is included in *dog*. In other words, while this statement affirms that every dog is a mammal, it does *not* affirm that every mammal is a dog.

Importantly, this is an assertion about the entire group of the subject, *dog*s, but *not* about the entire group of the predicate, *mammal*s. In other words, the predicate of affirmative propositions is *undistributed*.

Dogs are not reptiles

This is a negative proposition; it denies that the predicate *reptile* applies to the subject, *dogs*. It means that the whole extension of *reptile*, meaning every creature to which the term *reptile* applies, is not a *dog*. However, it does *not* mean that the whole comprehension of *reptile* does not apply to *dog*; for example, both dogs and reptiles are living, have blood, and so forth. So while the proposition denies that dogs are reptiles, it does *not* deny all similarity between dogs and reptiles.

Importantly, this contains an assertion about the entire group of the predicate *reptile*s *and* an assertion about the entire group of the subject, *dog*s. In other words, the predicate of a negative proposition is *distributed*.

Do not be fooled by mere form! Just because the sentence does or does not include the word "not" does not decide whether it's affirmative or negative. Consider the following:

All horses are not birds

No horses are birds

Which of these is affirmative and which negative, if either? The answer is that *both are negative*, even though only one contains a negative verb. In both cases, the predicate is *distributed*, taken in *all of its extension* but only *part of its comprehension*. Both statements mean that every creature to which the word *bird* applies is *not* a *horse*; but neither means that *bird* and *horse* have nothing in common. Look at the *meaning*, not merely the *form*.

The important things to remember from these considerations are the following:

1. *The predicate of a negative proposition is distributed.*
2. *The predicate of an affirmative proposition is undistributed.*

These two characteristics of affirmative and negative propositions become extremely important when we're applying the rules of reasoning.

Exercises 4-3

Tell whether the proposition is affirmative or negative. **1.** All hammers are tools Answer **2.** No hammers are screwdrivers Answer **3.** Hammers are not screwdrivers Answer **4.** Turtles are not birds Answer **5.** No turtles are not reptiles Answer

Subsection 4.1.4 Naming the Propositions

It's necessary to refer to propositions by all of these classifications at times; but it is *very often* necessary to refer to a proposition by its form—that is, whether it is *affirmative* or *negative*—and by its quantity—that is, whether its subject is *distributed* or *undistributed* together. E.g., we refer to a proposition as *universal affirmative*, or *particular negative*.

For these reasons, as well as for describing the forms of syllogisms later on, we label types of propositions in a certain way:

- A — *Universal affirmative*: an affirmative proposition predicating to *all* of the subject. E.g., *all men are rational.*
- E — *Universal negative*: a negative proposition predicating to *none* of the subject. E.g., *all men are not reptiles.*
- I — *Particular affirmative*: an affirmative proposition predicating to *some* of the subject. E.g., *some men are blond.*
- O — *Particular negative*: a negative proposition predicating to *some* of the subject. E.g., *some men are not blue-eyed.*

Committing these literal codes to memory will pay huge dividends down the line.

Exercises 4-4

Tell the type of proposition and its corresponding letter. **1.** Some men are dogs Answer **2.** All men are dogs Answer **3.** All men are not dogs Answer **4.** No men are not dogs Answer **5.** Some men are not dogs Answer **6.** No hammers are screwdrivers Answer **7.** No Catholics are Protestants Answer **8.** Some Protestants are not Christians Answer **9.** Some Christians are Protestants Answer **7.** Some computers are Sun workstations Answer **8.** No computers are horses Answer **10.** Dogs are cats Answer **11.** Men are apes Answer **12.** Some men are apes

Answer **13.** No slide rules are electronic Answer **14.** No computers are wooden Answer **15.** Some wood is pine Answer

Section 4.2
Complex Propositions

So far, we have seen only *simple* propositions; however, there are a number of types of complex propositions that we also need to consider.

Subsection 4.2.1 Conjunctive Propositions

Conjunctive propositions involve multiple subjects, multiple predicates, or both, each of which are joined together by a conjunction; in English, "and" or "nor".

conjunctive proposition

a proposition containing multiple subject and multiple attributes joined by an affirmative or negative conjunction; i.e., *and* or *nor*

These propositions are true only if each of its parts are true. It can be useful to split these propositions into their component parts. E.g.:

Dogs and cats are mammals

This proposition is only true if both dogs and cats are mammals. To make this strict logical form, separate it into two propositions:

Dogs are mammals

Cats are mammals

Each one can then be analyzed on its own.

Subsection 4.2.2 Disjunctive Propositions

These propositions state both an incompatibility and an alternative.

disjunctive propositions

a proposition stating both an incompatibility and an alternative

Disjunctive propositions are true only if the two parts are mutually opposed *and* there is no middle possibility. An example:

Actions are good or bad

Assuming the impossibility of an action which is neither good nor bad (an assumption we don't need to get into right now), we have a disjunctive proposition. An action is either good, or it is bad; and it cannot be neither.

Like conjunctive propositions, it is sometimes useful to split these propositions into their component parts, parts, *and remember the implied dichotomy*. For this example:

Some actions are good

Some actions are bad

No actions are neutral

No actions are both good and bad

So the total proposition, *actions are good or bad*, is true only if *at least one* of the first two propositions are true, *and* the last two are true.

Subsection 4.2.3 Conditional Propositions

Conditional propositions have two parts, connected by the word *if*. The first is the *antecedent*; the second is the *consequent*.

conditional propositions

double propositions in which the truth of the second (the consequent) depends upon the truth of the first (the antecedent)

These types of propositions are true when the *consequence* (not to be confused with that of the *consequent*) is true; the truth or falsity of the parts themselves is immaterial. For example:

If the soul is spiritual, it is immortal

Here we have two propositions in a conditional relationship. We can separate them thus:

The soul is spiritual

The soul is immortal

This is true if the *consequence* is true; namely, if the fact of being spiritual would make the soul immortal. In other words, if the following is true:

All spiritual things are immortal

It is, therefore, totally irrelevant to the truth of the proposition if both *the soul is spiritual* and *the soul is immortal* are false. The *consequence* is true; and that is what matters.

Subsection 4.2.4 Causal Propositions

Causal propositions, as the name implies, express a *causation*, and are really two propositions combined into one.

causal propositions
a proposition which contains two others, the first expressed as a cause of the second

These propositions are true only if both propositions are true, *and* the first is really the cause of the second.

Subsection 4.2.5 Relative Propositions

Relative propositions express a connection between two terms; e.g., *As the life is, so the death is*. They are true if the connection between the two is true.

Subsection 4.2.6 Adversative Propositions

Adversative propositions are multiple propositions separated by something such as "but", "yet", or "nevertheless".

adversative propositions
multiple propositions set into opposition by conjunction such as "but", "yet", or "nevertheless"

Much like relative propositions, these are true only if their constituent propositions are true *and* the opposition between them is true.

Subsection 4.2.7 Exclusive Propositions

These propositions assert that a predicate belongs not only to the subject, but to that subject *alone*.

exclusive propositions

propositions assigning a predicate to a single subject and to no other

This really two propositions combined into one. Take, as an example, the proposition *God alone is eternal.* This is truly the following two propositions:

God is eternal

No not-God is eternal

The source proposition is only true if both constituent propositions are true.

Subsection 4.2.8 Exceptive Propositions

These propositions affirm an attribute of a subject, but except certain subdivisions of that subject. In reality, as is often the case with complex propositions, this is actually two or more propositions combined into one.

exceptive propositions

a proposition affirming an attribute of a subject, but excepting certain subdivisions of that subject

Take as an example the proposition *Mammals have fur, except dolphins* (which is not true, but for our purposes that doesn't matter). This particular exceptive proposition is really three propositions:

Some mammals have fur

Dolphins are mammals

Dolphins do not have fur

These propositions are true only if *all* the constituent propositions are true.

Subsection 4.2.9 Comparative Propositions

These propositions say not only that a thing is so, but that it is more so; less so; or equally so as another thing.

> ***comparative propositions***
>
> propositions not only affirming a thing, but also affirming it to a greater, lesser, or same degree than another thing

These are multiple propositions combined into a single proposition. Consider the comparative proposition that *mammals are warmer than birds*. This includes the following propositions:

Mammals are warm

Birds are warm

The warmness of mammals is larger than the warmness of birds

These are true only if *all* of these constituent propositions are true. In this case, not, because on average the warmness of birds is actually greater than that of mammals.

Subsection 4.2.7 Inceptive Propositions

These propositions indicate that something *began* or *ceased* at a certain point:

> ***inceptive propositions***
>
> propositions affirming not only the existence of something, but also its beginning or ending

These are all two propositions. For example, *The United States became independent in 1776* is really these two propositions:

The United States is independent

That independence happened in 1776

It is true only if *all* of its constituent parts are true.

Section 4.3
Relations Between Propositions

Propositions may be related by *equivalence*, *convertibility*, *subordination* and *opposition*.

Subsection 4.3.1 Equivalent Propositions

Propositions are *equivalent* when they differ only in expression, but have the same sense and logical value. For example:

> *All men are mammals*
>
> *No men are not-mammals*

While these two propositions are not *identical*, in the sense that they are phrased differently, they have exactly the same meaning; namely, that each and every man is a mammal.

Subsection 4.3.2 Convertibility of Propositions

Sometimes, one proposition can be converted into another proposition, so that the truth value of both the original and the new shall be the same. Note that this isn't *equivalence*; the new proposition will have a different meaning than the original. But if the original is true, the new one will still be true.

Three types of propositions are convertible into new, also true propositions. Convertibility depends upon the extension of the terms in the proposition.

Universal negative propositions can be converted, because both terms, the subject and the predicate, are universal (that is, *distributed*). (Remember that it's universal because its subject is distributed; and since it's negative, its predicate is also distributed.) For example:

> *No mineral can speak*
>
> *Nothing that speaks is a mineral*

Particular affirmative propositions are convertible, because both terms are undistributed. (Remember that it's particular because its subject is undistributed; and since it's affirmative, its predicate is also undistributed.) For example:

> *Some animals have reason*
>
> *Some reason-havers are animals*

Universal affirmative propositions are convertible, *if the predicate of the first is undistributed in the second.* Of course, the predicate of an affirmative proposition is undistributed anyway; but we must specifically mark the subject of the resulting proposition as undistributed, perhaps with the adjective "some", to make it clear what were actually asserting. For example:

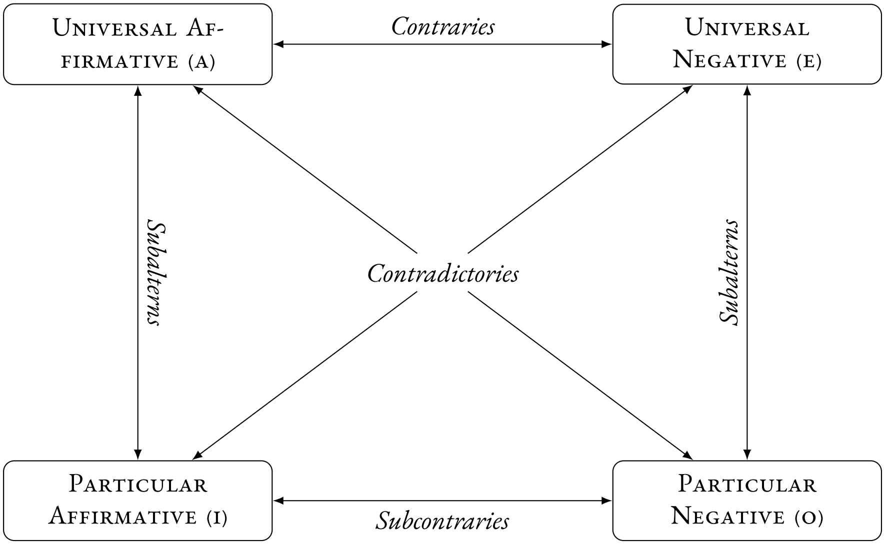


Figure 3: *Square of Opposition*

All men are rational

Some rational things are men

It's very important to remember this; many people are led to false conclusions by attempting to convert universal affirmative propositions without adding this particularizing quantifier.

Exercises 4-5

Convert the following propositions, or state if they are not convertible. **1.** Some animals are not rational Answer **2.** All men are animals Answer **3.** No men are women Answer **4.** Some hammers are not tools Answer **5.** Some hamers are tools Answer

Subsection 4.3.3 Opposition

Propositions are, of course, often opposed to one another. Logicians have distinguished between four types of opposition, which are represented graphically in Figure 3.

Contradictories: Some propositions are so totally opposed to one another that they exclude any intermediate judgment whatsoever; these are said to be *contradictory*. They differ in both form and quantity; so a and o propositions are contradictories, as are e and i propositions. For example:

All men are rational

Some men are not rational

This is an A and an O. There is clearly nothing in common between these two; if one is true, than the other is wholly false, and vice-versa; and moreover, there is no middle proposition that might be true if both of these are false.

All men are not rational

Some men are rational

This is an E and an I. Once again, there is no middle ground here; if one is true, the other is wholly false, and vice-versa. Contradictories can never be both true or both false; they must have different truth values.

Contraries: Propositions which have different forms, but universal quantities, are opposed to each other but do not exclude any intermediate judgment. These are called *contraries*. For example:

All men are just

All men are not just

This is an A opposed to an E. These cannot both be true, of course; but there *is* a middle ground that might be true; namely, that *some men are just*.

In other words, contraries cannot both be true, but they may both be false. Logicians say that *the falsity of a proposition does not imply the truth of the contrary*. The contrary might also be false, and the middle judgment—in this case, that *some men are just*—be true.

Subcontraries: propositions which differ in form, but are both particular in quantity, are called *subcontraries*. For example:

Some men are just

Some men are not just

This is an I and an O. Both of these propositions, of course, can be true; however, both cannot be false. In this case, if *some men are just* is false, then *all men are not just*, the contradictory, must be true; if *all men are not just* is true, then *some men are not just* is also true. So while both can be true, *subcontraries cannot both be false*.

Subalterns: Propositions with the same form, but different quantity, are *subalterns*. For example:

All men are just

Some men are just

This is an A and an I. These two are barely opposed at all; one just expands farther than the other, and both can be true.

The truth of a universal implies the truth of the subaltern, and *the falsity of a subaltern implies the falsity of the universal.* Neither of these holds in reverse.

These rules enable us to do some *immediate inferencing* when we're confronted with a proposition.

> ***immediate inference***
>
> conclusions that can be drawn from a single proposition

As we shall see when we reach Chapter 5, typically we require *two* propositions to come to a conclusion; the nature of opposition, however, enables us to draw some conclusions from single propositions, which we call *immediate inferences.*

For example, when given a proposition, we can immediately infer the following:

1. Its contradictory must be *false*.
2. Its contrary may be false, if it itself is false; but if it is true, its contrary must be false.
3. If it is false, its subcontrary is true.
4. If it is a universal proposition, and it is true, then its subaltern is also true.
5. If we know that its subaltern is false, and it is a universal, then it is also false.

Just learning these rules of relations and opposition, then, can help us learn a great deal, before we even must begin to engage in formal reasoning.

Exercises 4-6

Give the type of opposition between the propositions. **1.** All men are apes; no men are apes Answer **2.** All men are apes; some men are not apes Answer **3.** All hammers are not tools; some hammers are not tools Answer **4.** All turtles are reptiles; some turtles are not reptiles. Answer **5.** Some turtles are not reptiles; some turtles are reptiles Answer **6.** All hammers are tools; some hammers are tools Answer **7.** Some hammers are not screwdrivers; all hammers are not screwrivers Answer **8.** Some computers are not useful; all computers are useful Answer

Tell what immediate inferences can be drawn from the following propositions, assuming the truth value asserted. **9.** All men are apes; true Answer **↊.** No men are reptiles; true Answer **↋.** Some men are reptiles; false Answer

Section 4.4
Strict Logical Form

It is sometimes helpful to rephrase a proposition into *strict logical form*, to help one better see the essential characteristics of the proposition: namely, whether it is positive or negative, and whether it is distributed or undistributed.

Strict logical form

a sentence structure designed to make affirmativity or negativity and distribution clearer than natural language

Strict logical form involves rephrasing a normal proposition into an often stilted, bizarre-sounding pseudo-grammatical English that only a logician could love. But again, rearranging the sentence in this way can sometimes make it easier to spot fallacies and otherwise verify the correctness of reasoning done on propositions.

Be careful not to change the meaning of the proposition by putting it in strict logical form.

First, arrange the sentence so that the verb is *always* a form of "to be", typically "is" or "are". This means that active or passive verbs need to be changed to agentive nouns in the predicate. Take the proposition

> *horses eat grass*

In strict logical form, the verb has to be "are", so this would have to be changed to:

> *horses are eaters-of-grass*

But we also need to make sure that the *distribution* of the proposition is explicit, so we need to add the word "all" or "some" to the subject. In this case, we evidently intend the subject to be distributed, so we use "all":

> *all horses are eaters-of-grass*

If we intended the subject to be undistributed, we would use "some"; for examples: *some horses are black*.

To clarify whether the proposition is affirmative or negative, we need to turn an exclusive subject into an inclusive one. For example, when we say

> *no horses are meat-eaters*

we need to rephrase that. Remember that when we say *none* we are using the term in a distributed way; but we're still using it in a negative way. But that negative nature may be concealed by the use of an affirmative verb. So we should rephrase this:

> *all horses are not meat-eaters*

Oftentimes, as noted above, putting a phrase in strict logical form makes it sound extremely odd. A singular subject, for example, is distributed; but strict logical form requires the distributed nature of the subject to be explicit. So we must use “all”. So a proposition like the following:

> *Rover fetches the frisbee*

has to become the very awkward:

> *All Rover is a frisbee-fetcher*

Obviously, we would never actually speak this way; but when we’re analyzing whether a proposition is being used correctly, and whether we’re reasoning correctly with it, be aware that strict logical form is an option to help determine compliance with the rules that we are going to observe shortly.

Chapter 5
Deductive Reasoning

Reasoning is the entire point of all the study we've made so far. Certain things we can know *immediately*, simply by seeing them; but certain things we can only know *mediately*, by reasoning from one proposition to another. Knowledge had immediately is called a *principle*; of this nature are the fundamental principles we looked at in Chapter 2, such as the principle of non-contradiction and the principle of sufficient reason. Those truths which are arrived at mediately are called *conclusions*. To proceed from principles to conclusions is called *reasoning*.

reason

the process from known premises to unknown conclusions

The fundamental form of reasoning is called the *syllogism*, in which two propositions are used to reach a third proposition. The first two propositions are called the *premises*; the last is called the *conclusion*.

syllogism

the verbal expression of an act of reasoning, in which two premises yield a conclusion

It's important to note that, when we're studying logic, we're less concerned with *truth* than we are with *validity*. The goal of logic is, of course, to ensure that we can reach true conclusions; but when we're studying logic itself, we're concerned with *using valid arguments*.

It's possible to have true premises but use bad reasoning to come up with a false conclusion. It's possible to use bad reasoning and yet still get a true conclusion; but it will be true by happenstance, not because of good reasoning. It's also possible to use good reasoning but still get a false conclusion—if the premises are false. But when we're studying logic, we're not worried about whether our conclusion is true; we're worried about whether our reasoning is valid. When you're using logic in the real world, absolutely make sure that your premises are true, and use reasoning to draw a true conclusion; but when you're studying logic, worry about whether your reasoning is valid, not whether your conclusion is true. We'll have many goofy conclusions in this text that nevertheless demonstrate good reasoning.

Keep in mind that, when we show something is true from known premises and valid reasoning, we have *demonstrated* it. These *demonstrations* give us absolute certainty of the truth of our conclusions; we have no reasonable fear that we might be wrong about them. When our premises are merely probable, we have demonstrated only a *conditional* certainty; that is, conditional on the truth of our premises. That is called *opinion*; we believe something, but we recognize that it might be false. This is an important distinction to keep in mind, especially when engaged in disputes about given conclusions.

Section 5.1
Terminology of the Syllogism

A syllogism must contain three and only three terms. Each premise contains two terms, a subject and a predicate; one of those terms must be common between the two. This term, common between both premises, is the *middle term*, while the other terms are the *extremes*, broken up into the *major* and *minor*.

middle term

the term common to the two premises of a syllogism

extreme term

the term of a premise which is not common with the other premise

major term

the extreme term which will form the predicate of the conclusion

minor term

the extreme term which will form the subject of the conclusion

The extremes are sometimes differentiated between the *great extreme*, which is the extreme term in the *major premise*; and the *small extreme*, the extreme term in the *minor premise*. But it is usually more helpful to think of them as the major and minor term, rather than the great and small extreme.

Which brings us to the terms *major and minor premise* themselves. The *major premise* is that premise which compares the major term to the middle term; the *minor premise* is

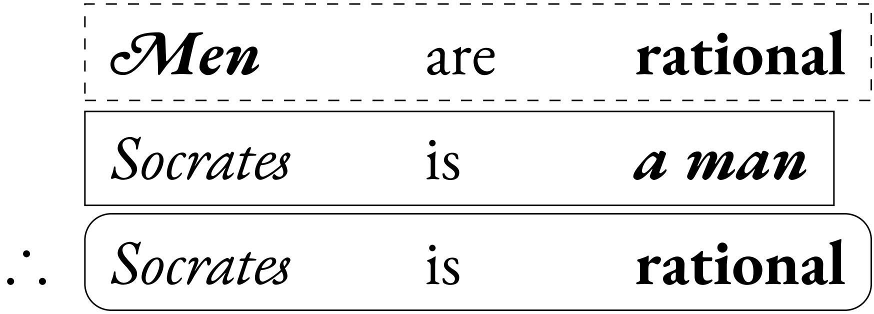


Figure 4: *Parts of a Syllogism*

that premise which compares the minor term to the middle term. Together, they form the *antecedent*.

> ***antecedent***
>
> the major and minor premises of a syllogism

It's important to note that the major premise is not necessarily the premise which comes first; rather, it's premise that contains the *major term*. The major term is the term which will form the predicate of the conclusion, while the minor term will form the subject of the conclusion.

From the antecedent we draw our *conclusion*:

> ***conclusion***
>
> a proposition linking the great and small extremes, drawn from the major and minor premises; sometimes also called the consequent

This mouthful of terminology will now get us where we need to go: a systematic study of the syllogism. For a more visual representation, consider Figure 4.

We can see in this figure the following parts:

- ***Bold and italic*** is the all-important middle term, which connects the two premises and enables us to draw a conclusion.
- **Bold** is the great extreme, or major term.
- *Italic* is the small extreme, or minor term.
- The dashed rectangle surrounds the major premise.
- The solid rectangle surrounds the minor premise.

- The rounded rectangle surrounds the conclusion.

The symbol "∴" is pronounced "therefore", and has that meaning.

Section 5.2
The Eight Rules of the Syllogism

The syllogism, the paradigmatic act of reasoning, is governed by rules, as is everything else in logic; those rules are essentially eight.

Rule 1: There must be three and only three terms The major term, minor term, and middle term must all be distinct from one another, and there must be no additional terms. Remember, too, that this means three *in meaning*, not merely three words. Equivocation—that is, using the same word in more than one way—is the most common cause of violating this rule.

Rule 2: No term of the conclusion can have a greater extension than it has in the premises This is because we cannot extrapolate from *some* of a thing to *all* of it; it's the same reason we can't look at a group of Swedish skiers and conclude that all humanity is blond. *Some* humans are blond; but we cannot speak about *all* humans' hair color based on that.

All sheep eat grass
Horses are not sheep
∴ *Horses do not eat grass*

This is a clearly false conclusion, despite the fact that both premises are clearly true. The falsity is because *eat grass* in the major premise is *undistributed*, or not extended (which we know because it's the predicate of an affirmative proposition); while in the conclusion *eat grass* is *distributed*, or extended (which we know because it's the predicate of a negative proposition); that is, I'm speaking of the whole class of grass-eaters, and excluding horses from it.

Violating this rule is called an *illicit process*; for more about which, see Section 5.5.4.

Rule 3: The middle term must not appear in the conclusion Simply put, the syllogism connects the major and minor terms by means of the middle to make a conclusion; the middle is the means, not the end.

Rule 4: The middle term must be distributed in at least one of the premises The major and minor terms are compared by means of the middle term. But if the middle term is not distributed in at least one of the premises, we can't be sure that we're talking about the same part of the major and minor terms in order draw a conclusion.

Some learned men are unbelievers
The Doctors of the Church were learned
∴ *The Doctors of the Church were unbelievers*

The falsity of the conclusion is evident; but what is the falsity of the reasoning? The middle term is undistributed in both premises. *Learned* is undistributed in the major premise (we can clearly see that by the use of the word "some"), and it is also undistributed in the minor premise (clearly, we're not asserting that the Doctors of the Church encompass *all* learned men, but merely that they are included in that group). Since both are undistributed, it's entirely possible (and indeed, here, it is entirely true) that the major premise is talking about an entirely different part of the group "learned men" than the minor premise is discussing. So no conclusion can be drawn.

Violating this rule is the famous *undistributed middle*, which we discuss further in Section 5.5.3.

Rule 5: From two negative premises no conclusion can be drawn We need at least one affirmative premise; otherwise, we're not affirming anything about the middle term. By denying anything about the middle term, we can't connect the major and minor terms by means of it; but reasoning connects major and minor terms by means of middle terms, so this fact makes reasoning impossible.

Rule 6: From two affirmative premises a negative conclusion cannot be drawn If we're affirming some agreement of both the major and minor terms with the middle term, then we are *affirming*, not denying, something about the agreement between the major and minor terms. So we cannot have a negative conclusion.

Rule 7: No conclusion can be drawn from two particular premises This rule is very similar to the rule of distributed middles. Indeed, if both premises are affirmative and particular, then the middle term will be undistributed in both premises, and no conclusion can be drawn; and it is exactly the same as the rule of distributed middles.

If one of the premises is negative and one affirmative, then there are two possibilities. In both possibilities, since affirmative propositions have undistributed predicates, we must look to the negative proposition for the distributed middle term.

First, if the negative proposition is the major term, then its undistributed subject will be the predicate of the conclusion. However, the conclusion must be negative (since one of the premises is negative), and negative propositions have distributed predicates. Therefore, this combination will result in a term being distributed in the conclusion but undistributed in the premise, which violates Rule 2.

Second, if the negative proposition is the minor term, then its undistributed subject will be the subject of the conclusion. However, the conclusion must be negative (since one of the premises is negative), which means that its predicate will be distributed. Its predicate, however, comes from the major premise, which must be affirmative; but we've already seen that our affirmative premise's subject is undistributed (since it's a particular premise), and we know that the predicate of an affirmative premise is also undistributed. Either way, we have a term distributed in the conclusion but undistributed in the premise, which violates Rule 2.

So from two particular premises, no conclusion can be validly drawn.

Rule 8: The conclusion must follow the weaker premise This rule means that the conclusion must be particular if either premise is particular, and negative if either is negative.

To sum up the rules of the syllogism:

1. There must be three and only three terms.
2. No terms can have greater extension in the conclusion than in the premises.
3. The middle term cannot appear in the conclusion.
4. The middle term must be distributed in at least one of the premises.
5. From two negative premises, no conclusion can be drawn.
6. From two affirmative premises, only an affirmative conclusion can be drawn.
7. No conclusion can be drawn from two particular premises.
8. The conclusion must follow the weaker premise.

Exercises 5-1

Based on the eight rules of the syllogism, tell whether the following syllogisms are valid; and if not, which of the eight rules they are violating.

1. Disobedient children must be punished; little Timmy is a disobedient boy; ∴ little Timmy must be punished Answer

2. Good oratory is genius; Cicero was a great orator; ∴ Cicero's genius was in his great oratory Answer

3. Jews hate Christians; Joe is a Jew; ∴ Joe hates Christians Answer

4. All ostriches are rocks; all rocks are monkeys; ∴ all ostriches are monkeys Answer

5. Shoemakers are not astronomers; astronomers are not gophers; ∴ shoemakers are not gophers Answer

6. All pages wear their master's colors; all books are made up of pages; ∴ all books wear their master's colors Answer

7. Some cabbies are rude; some men are cabbies; ∴ some men are rude Answer

8. All jewels are minerals; all diamonds are jewels; ∴ all diamonds are minerals Answer

9. All lemons are sour; some fruits are lemons; ∴ some fruits are not sour Answer

ꝛ. Buffalo are fierce; tigers are not buffalo; ∴ tigers are not fierce Answer

Section 5.3
Figures of the Syllogism

Syllogisms appear in multiple *figures*, which are determined by the position of the middle term with respect to the two extremes. There are four possible figures which can result in a logical conclusion, each of which we will examine in turn.

In the diagrams on this matter, Ma will represent the major term, Mi the minor, and M the middle.

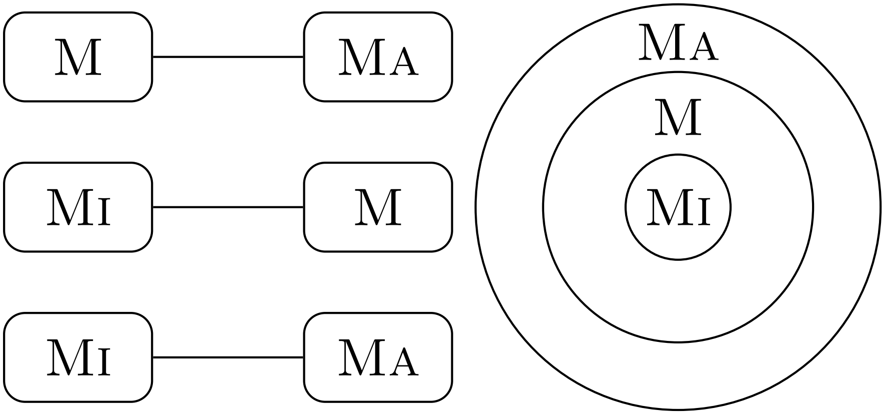


Figure 5: *The First Figure*

Also, if we look back to Section 4.3.3, we will remember our types of propositions, A, E, I, and O. These abbreviations for different types of propositions—universal affirmative, universal negative, and so on—become very convenient for labelling syllogisms when we group them in threes; namely, one for the major premise, one for the minor, and one for the conclusion. So AAA would represent a syllogism with an A for the major premise, an A for the minor premise, and an A for the conclusion. We will use this notation very frequently in our discussions ahead.

It's useful to memorize these possible combinations for each figure; indeed, it's *de rigeur* when studying logic to do so. However, it's important not to reduce logic to applying these three letters to syllogisms. It's a good shorthand, but one should always be able to explain *why* a given figure works or does not work, rather than merely saying that it's not one of the correct combinations.

Subsection 5.3.1 First Figure

The first figure is that in which the middle term is the subject of the major premise and the predicate of the minor. It is the most basic and most common form of the syllogism, and consequently the one which is the easiest for us to see as rational.

In this figure, the middle term is a class which is greater in extension than the minor term, and less in extension than the major. For this reason, the rules of the syllogism are easiest to apply, and the figure most obviously appeals to our sense of reason. The other figures all depart from this in some way, and consequently are more difficult to arrange logically.

Think of the first figure as applying a general law to a particular case; the major premise

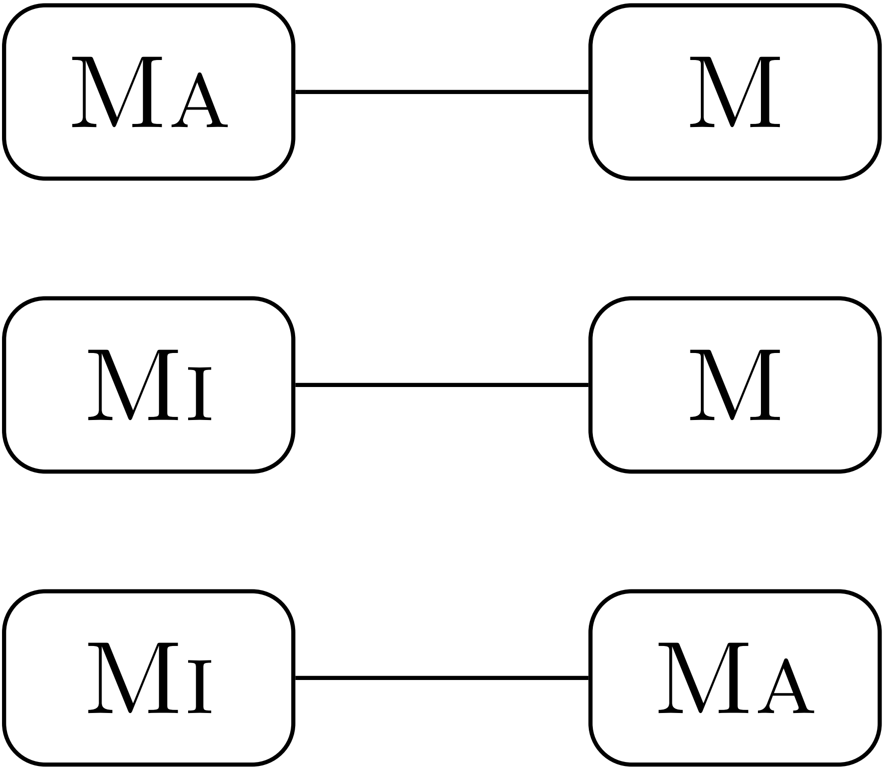


Figure 6: *The Second Figure*

states the law, and the minor premise applies it. The rules of the first figure are thus only two:

1. The major premise (which states the law) must be universal.
2. The minor premise (which applies the law) must be affirmative.

The valid syllogisms of the first figure are AAA, EAE, AII, EIO, AAI, and EAO (though the last two are just weaker versions of the first two). Any other combination will cause an error.

Subsection 5.3.2 Second Figure

In the second figure, the middle term is the predicate of both the major and minor premises.

In the second figure, one of the premises must be negative. In the affirmative premise, the middle term is either less extended than the major term or more extended than the mi-

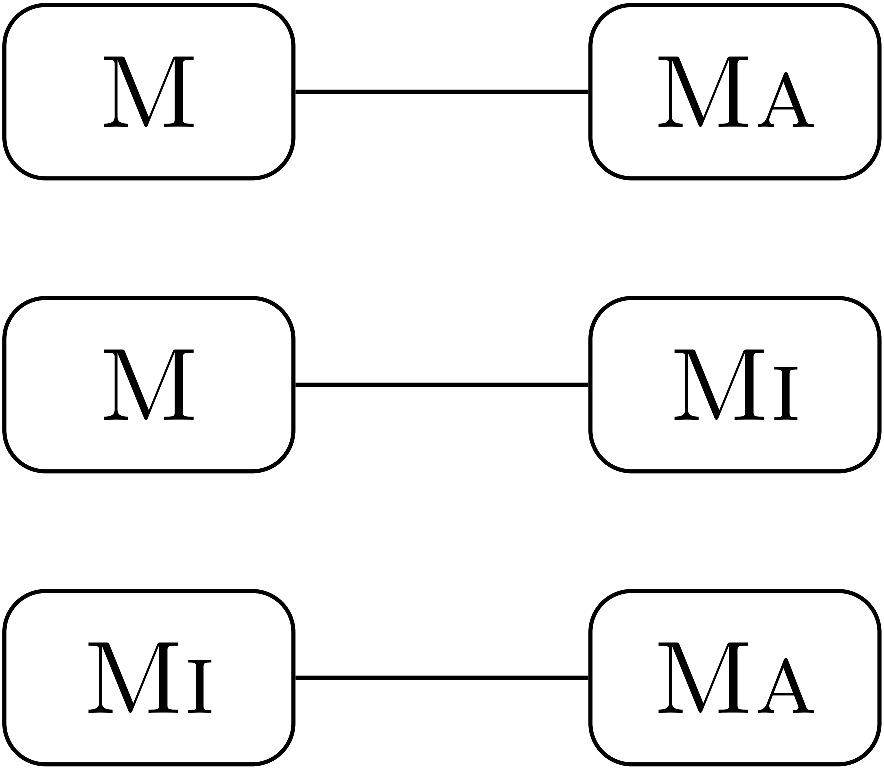


Figure 7: *The Third Figure*

nor; however, in the negative premise, it may not occupy its normal position of extension. The rules here are thus:

1. The major must be universal.
2. One premise must be negative.
3. The conclusion must be negative.

Only four possible syllogisms will work in the second figure: EAE, AEE, EIO, AOO.

Subsection 5.3.3 Third Figure

If the middle term is the *subject* of both premises, we have the *third figure*.

In the third figure we're taking the middle term in only *part* of its extension in one premise and *all* of it in the other. In this figure, the middle term is less in extension than both major and minor terms. We have thus the following rules:

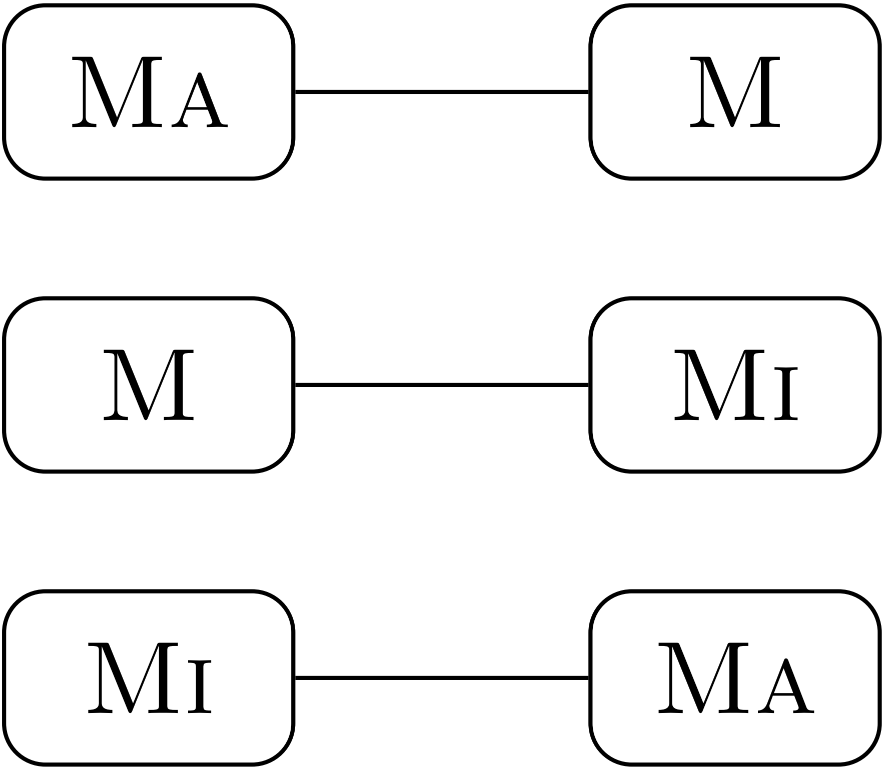


Figure 8: *The Fourth Figure*

1. The conclusion must be particular.
2. The minor premise must be affirmative.

The possible syllogistic forms here are AAI, IAI, AII, EAO, OAO, and EIO.

Subsection 5.3.4 Fourth Figure

If the middle term is the predicate of the major premise and the subject of the minor, we have the fourth figure.

This figure only works if we make the extension of the minor term larger than that of the major term, and the middle's extension in between. In this way it's the exact reverse of the first figure; it is thus the most opaque of the figures, and the least useful. Its rules are:

1. If the major is affirmative, the minor must be universal.
2. If the minor is affirmative, the conclusion must be particular.

3. If one premise is negative, the major must be universal.

The forms AAI, AEE, IAI, EAO, and EIO will work with this figure.

This figure is so seldom useful that Aristotle and the original scholastics didn't really consider it at all. Galen, a physician of the second century, worked out its rules; and thus one will sometimes hear it called the Galenian figure. Few others can make very good use of it.

Subsection 5.3.5 Summary of the Figures of the Syllogism

The four figures of the syllogism can be summed up as follows:

Figure	Major Premise	Minor Premise	Forms
First Figure	Subject	Predicate	AAA, EAE, AII, EIO
Second Figure	Predicate	Predicate	EAE, AEE, EIO, AOO
Third Figure	Subject	Subject	AAI, IAI, AII, EAO, OAO, EIO
Fourth Figure	Predicate	Subject	AAI, AEE, IAI, EAO, EIO

The second, third, and fourth figures are never really *necessary*, and only work insofar as we can reduce them to the first figure; we use them not because we need them, but because their expression is sometimes convenient. Indeed, any syllogism may be reduced to the first figure; and more, to AAA or EAE, if really pressed. It is not important, for our purposes here, to go through all the rules for this reduction; careful thought will reveal them, if they are really necessary. It is merely needed to note that it is possible.

There is a traditional rhyme, in mongrel mostly-pseudo Latin, to help remember the valid syllogistic figures. The capital letters in each word indicate valid proposition-type combinations:

> bArbArA, cElArEnt, dArII, fErIOque, prioris.
> cEsArE, cAmEstrEs, fEstInO, bArOkO, secundæ.
> Tertia, dArAptI, dIsAmIs, dAtIsI, fElAptOn,
> bOkArdO, fErIsOn, habet; Quarta insuper addit,
> brAmAntIp, cAmEnEs, dImArIs, fEsApO, frEsIsOn.

Nonsense words, however, are always difficult to keep straight, particularly in a second language. Those not conversant with Latin will find the above lines difficult to keep straight. The author suggests the following lines to keep these things straight, which has the added benefit that each line is a different figure:

> An *a*v*a*t*a*r *e*l*a*t*e*s the *a*c*i*d*i*c w*ei*rd*o*;
> an *ea*t*e*r *a*w*ee*k, in a r*e*g*io*n t*a*b*oo*,
> uses *a*b*aci* *i*nl*ai*d with ch*e*ap*o* *a*l*i*b*i*s; the p*erio*d of a

pot*ato*
and a leg*io*n of *ia*mb*ic* s*a*lam*i* *agree* in *seaso*n

Equally nonsensical (though at least all of these are actual words), but perhaps a tad easier to remember. Of course, both rhymes are entirely optional; if the student does not find them helpful, he should forget both.

Exercises 5-2

Tell what figure the syllogism is; give its three-letter signature; and tell whether it's valid or not.

1. Disobedient children must be punished; little Timmy is a disobedient boy; ∴ little Timmy must be punished Answer **2.** All oysters are nutritious; no oysters are in season in July; ∴ nothing in season in July is nutritious Answer **3.** Good oratory is genius; Cicero was a great orator; ∴ Cicero's genius was in his great oratory Answer **4.** No singers are virtuous; all actors are virtuous; ∴ no actors are singers Answer **5.** Jews hate Christians; Joe is a Jew; ∴ Joe hates Christians Answer **6.** All turtles are reptiles; no birds are reptiles; ∴ no birds are turtles Answer **7.** All ostriches are rocks; all rocks are monkeys; ∴ all ostriches are monkeys Answer **8.** All oysters are nutritious; all oysters are in season in September; ∴ some things in season in September are nutritious Answer **9.** Shoemakers are not astronomers; astronomers are not gophers; ∴ shoemakers are not gophers Answer **↊.** All pages wear their master's colors; all books are made up of pages; ∴ all books wear their master's colors Answer **↋.** Some pagans are virtuous; no burglars are virtuous; ∴ some burglars are not pagans Answer **10.** Some cabbies are rude; some men are cabbies; ∴ some men are rude Answer **11.** No mosquitos are pleasant; all mosquitos buzz; ∴ no buzzing things are pleasant companions Answer **12.** All jewels are minerals; all diamonds are jewels; ∴ all diamonds are minerals Answer **13.** All lemons are sour; some fruits are lemons; ∴ some fruits are not sour Answer **14.** All sparrows are impudent; some schoolboys are impudent; ∴ some schoolboys are sparrows Answer **15.** No mosquitos are pleasant; all mosquitoes buzz; ∴ some buzzing things are not pleasant Answer **16.** Buffalo are fierce; tigers are not buffalo; ∴ tigers are not fierce Answer **17.** All ostriches are birds; all birds can fly; ∴ some flying things are ostriches Answer

Section 5.4
Types of Syllogisms

All syllogisms are either simple or compound (also called *categorical* or *hypothetical*). So far, we have discussed only *simple* syllogisms; that is, those that consist of three simple propositions.

simple syllogism

a syllogism consisting only of three simple propositions

However, there are also *compound* syllogisms, which can consist of complex propositions (for the types of which see Section 4.2). We can always reduce complex syllogisms into simple syllogisms by breaking up compound propositions into their component parts; but very often that's not necessary, and the rules of dealing with compound syllogisms are explored here.

Subsection 5.4.1 Conditional Syllogisms

In a conditional syllogism, the major premise is a conditional proposition, and the minor either affirms the antecedent or denies the consequent.

conditional syllogism

a syllogism the major premise of which is a conditional proposition, and the minor either affirms the antecedent or denies the consequent

The conclusion will then be an assertion of the consequent or a denial of the antecedent, depending on the minor premise.

It's important not to deny the antecedent and thereby deny the consequent, or to affirm the consequent and therefore affirm the antecedent; this is a fallacy called *affirming the consequent* or *denying the antecedent* (see Section 5.5.5), and is a common source of errors in reasoning.

The rules of conditional syllogisms (in addition to those we already know for simple syllogisms) are three:

1. If we affirm the antecedent, we may affirm the consequent.
2. If we deny the consequent, we may deny the antecedent.
3. We may draw no conclusion from either affirming the consequent or denying the antecedent.

Take note, too, that the minor premise's affirmance or denial of one of the clauses of the major may not be obvious. For example:

If the skeptics are right, Scripture is not inspired by God
But Scripture is inspired by God
∴ *Skeptics are not right*

The minor premise here denies the consequent; but despite being a denial, is an *affirmative* proposition. This is because the consequent is a *negative* proposition. Be on the lookout for such things.

Exercises 5-3

Tell whether these are valid conditional syllogisms, and why. **1.** If the wind is from the north, the weather gets cold; the wind is in the north; ∴ the weather is cold. Answer **2.** If the beast has fur, it is a mammal; but the whale does not have fur; ∴ the whale is not a mammal Answer **3.** If a man has the plague, he is in danger of death; but this man has the plague; ∴ he is in danger of death. Answer **4.** If a man has plague, he is in danger of death; but this man does not have plague; ∴ he is not in danger of death. Answer **5.** If a man has the plague, he is in danger of death; but this man is not in danger of death; ∴ he does not have the plague Answer **6.** If a beast is a reptile, it has scales; this creature has scales; ∴ this creature is a reptile Answer

Subsection 5.4.2 Disjunctive Syllogisms

If the major premise is a disjunctive proposition, we have a *disjunctive syllogism*, in which the minor premise will assert or deny the truth of one of the alternatives which the major premise proposes.

disjunctive syllogism

a syllogism the major premise of which is a disjunctive proposition, the minor affirming or denying one of the alternatives

For this reason we insisted, in Section 4.2.2, that a disjunctive proposition leave no intermediate possibility; that is, that it be an *exclusive*, not an inclusive, disjunctive. If one, the other, *or both* could be true, than we cannot draw a conclusion from it. So a well-formed disjunctive proposition cannot include "or both" as a possibility. Disjunctive propositions must be exclusive both of one another and of everything else; otherwise we cannot draw a conclusion, and doing so will be a fallacy of the *false dichotomy*.

If there are more than two options in the disjunctive proposition, and the minor affirms one, the conclusion may deny the rest; if the minor denies one, then we can affirm at least one of the others.

A special case of the disjunctive syllogism is the *dilemma*:

dilemma

a syllogism with a disjunctive major and a minor which shows how each alternative establishes a point

A perfect example is the dilemma of the test-taker:

I will pass the test or I will fail it
If I pass, I will be proud; if I fail, I will not longer need to worry about it
∴ *No matter what happens, I have reason to be glad*

People often try to pose dilemmas without really making sure that the alternatives in the major are exhaustive, or that the consequences of the minor are really indisputable. The reply, if there is one, should suggest itself rather readily.

Exercises 5-4

Tell whether the following are valid, and why; additionally, note if the syllogism is a dilemma. **1.** Either the sun moves round the earth, or vice-versa; the sun does not move round the earth; ∴ the earth moves round the sun Answer **2.** Either I am older than you, the same age, or young; I am not older than you; ∴ I am younger Answer **3.** This man either lives in Massachusetts, New England, or New Jersey; he lives in New England; ∴ he does not live in Massachusetts or New Jersey Answer **4.** This man either lives in New York, New Jersey, or Pennsylvania; he lives in Pennsylvania; ∴ he does not live in New York or New Jersey Answer **5.** Either I pray, or I work; if I do the former, I will lose my livelihood; while doing the latter will cost me my soul; ∴ I have no good choices Answer **6.** This man lives in Texas or Mexico; he lives in Mexico; ∴ he does not live in Texas Answer

Subsection 5.4.3 Enthymemes

The *enthymeme* is essentially a syllogism which turns on *probability* rather than certainty. For the purposes of this type of syllogism, probability does *not* mean the mathematical probability we learned in school (e.g., that the probability of rolling a six on a normal die is 1/6); but rather that it is something that is known to be true *for the most part*. E.g., *blond people are blue-eyed*; *children look like their parents*.

probability
in logic, a tendency to be true for the most part, even if not infallibly

A *sign*, like something that is probable, is an indication of another reality. It does *not* mean that the other reality to which the sign points *must* be there, only that it often is.

sign
an indication of another thing

Enthymemes have a premise which is either a *probability* or a *sign*; for example:

Men who stagger are drunk
This man is staggering
∴ *This man is drunk*

Of course, this isn't infallible—it may be that the man has a bad ankle and hasn't had a drop to drink—but it's useful. It identifies a tendency and reasons from it.

enthymeme

a syllogism which has as a premise either a probability or a sign

This sort of syllogism is often called *rhetorical*, because orators—people engaged in rhetoric—use this type of syllogism almost exclusively. Because orators often omit one of their premises (typically the obvious ones; things like, "prosperity is a good thing"), enthymemes are also often identified as syllogisms in which one premise has not been expressly stated.

Subsection 5.4.4 Epichirems

The *epichirem* is unremarkable, except that one of its premises contains the reason for its truth.

epichirem

a syllogism in which one of the premises contains the reason for its own truth

Typically, a syllogism takes no notice of the truth or falsity of its premises, and merely ensures that the conclusion drawn from them is valid. With the epichirem the premise asserts a reason that it is true. E.g.:

All rational beings are to be treated with respect

This is a proposition; whether it is true or not is to be determined elsewhere. In an epichirem, however, we might encounter the following version of that proposition:

All rational beings are made in the image of God, and are to be treated with respect

We draw no part of our conclusion from "are made in the image of God"; we are merely justifying our proposition as we use it.

It may be that our epichiremic proposition is uncontroversial, and we merely state it with its reason for rhetorical purposes. However, we can always break it up into two syllogisms if necessary; one which will prove the connection between the proposition and the reason for its truth, and then the one in which we are using the epichirem.

Subsection 5.4.5 Sorites

Sorites is nothing but a series of syllogisms strung together, the predicate of each proposition being the subject of the next, with the final conclusion linking the subject of the first and the predicate of the last.

All Christians are followers of Christ
Followers of Christ must do as Christ did
Doing as Christ did is helping the poor
∴ *All Christians must help the poor.*

Obviously, sorites can go on quite a bit longer than this one, but the notion is there. It's important to note that even a single error anywhere in the chain can undo the argument, so be quite careful that each premise really does follow.

Of course, a sorites can always be broken up into the same number of syllogisms as there are propositions between the first and the last; in our example above, two. The second proposition will be the major of the first syllogism, and the first proposition will be the minor. So, as above:

Followers of Christ must do as Christ did
All Christians are followers of Christ
∴ *All Christians must do as Christ did*

Then we take the third proposition as the major and the conclusion just drawn as the minor:

Doing as Christ did is helping the poor
All Christians must do as Christ did
∴ *All Christians must help the poor*

Because we break it up into syllogisms of the first figure, it must obey the rules of that figure. From this, we can determine that only the *first* premise can be particular, and only the *last* premise can be negative. Otherwise, our conclusion will not follow.

Sorites, then, is really just a shorthand; and we should likely break it up into syllogisms before employing it as an argument, to ensure that it really works.

Exercises 5-5

1. There is a logical error in the above sorites. Tell what it is. Answer

Section 5.5

Fallacies

When we make an error in our *reasoning*, we have committed a *fallacy*. A fallacious syllogism is not, strictly speaking, a syllogism at all, but a *paralogism*; but calling it a syllogism, which may or may not contain a fallacy, is perfectly correct.

fallacy
an error in the form of argument

This is distinct from just being wrong; that is, having false premises, which is a *sophism*.

sophism
an error in the matter of argument; that is, a false premise

A syllogism which contains a sophism is called, easily enough, a *sophistical syllogism*

Subsection 5.5.1 Four Terms (Quaternio terminorum)

Rule 1 states that a syllogism has three and only three terms. When we introduce a fourth, we can no longer link two terms by means of a middle term, and thus cannot draw a valid conclusion.

It's easy to look at a syllogism and determine whether there are only three *words*; it is less easy to determine whether there are only three *terms*. It is the *meaning*, and not the sequence of letters, that is important. Thus, a fourth term is typically introduced by means of *equivocation*; that is, by using one term with two different meanings.

Man is free
Free creatures can do as they will
∴ *Man can do as he will*

When we say that man is *free*, we don't mean the same thing as when we say that freedom means we can do we will. One refers to *free will*, one refers to mere *ability*. This means we have four terms, and our conclusion is invalid.

This fallacy may also be committed by the use of *ambiguous* terms; that is, terms the exact meaning of which is unclear. This is a sort of uncertain equivocation. The use of *free* in our previous example is a good illustration of ambiguity.

Subsection 5.5.2 Begging the Question (Petitio Principii)

This is essentially a violation of the three-term rule, as well; but in this case there are one too *few* terms, rather than one too many. "Begging the question" has largely lost its technical meaning in common parlance; it is typically used to mean, "That makes me wonder", or "that neglects this other question". In logic, though, it means something very particular.

Another name for this fallacy is *circular argument*.

"Begging the question" means *using the conclusion to prove itself*; it's when the premises assume the truth of the conclusion. An informal way of explaining it is to say that *it answers the question by the question*.

Everyone wants this toy, because it's the hottest toy on the market!

This argument begs the question; the whole reason that it's the hottest toy on the market is that everyone wants it. The argument assumes its premise.

The *vicious circle* is a form of begging the question; it not only assumes what it is proving, but proves a second proposition by the first, then the first by the second.

Subsection 5.5.3 Undistributed Middle (Non distributio medii)

Rule 3 states that the middle term must be distributed in at least one of the premises. If it is undistributed in both terms, we have the fallacy of the *undistributed middle*.

Every metal is heavy
This iceberg is heavy
∴ *This iceberg is a metal*

Obviously false; the iceberg is *not* metal. But both the premises are true; therefore, we have made some error in logic. The error is that the middle term is undistributed in both the major and minor premises.

Metals truly *are* heavy, and icebergs are likewise heavy. But only *some* heavy things are metal; some heavy things are *not* metal. Since the middle term, *heavy things*, may exclude either the major or minor terms, we can't draw a valid conclusion.

Subsection 5.5.4 Illicit Process

We already saw, in Section 5.2 (specifically our discussion of Rule 2), that no term in the conclusion can have a greater extension than it has in the premises. We saw a particular example of that:

All sheep eat grass
Horses are not sheep
∴ *Horses do not eat grass*

Here, *eat grass* is the major term; it is undistributed in the major premise (since it's the predicate of an affirmative proposition). But in our conclusion, *eat grass* is distributed (since it's the predicate of a negative proposition); our conclusion covers a greater extension than our premises, which cannot work. Because our major term has overextended itself, we call this an *illicit process of the major*.

The same thing can happen with the minor term. For example:

Occasions of sin must be avoided
Drinking alcohol is an occasion of sin
∴ *Drinking alcohol must be avoided*

Our minor term here, *drinking alcohol*, is *undistributed*, because not *all* alcohol consumption is an occasion of sin. Yet in the conclusion, *drinking alcohol* is clearly taken in its full extension. There has been, then, an *illicit process of the minor*.

Subsection 5.5.5 Affirming the Consequent or Denying the Antecedent

As we've seen above (in Section 4.2.3, on conditional propositions; and in Section 5.4.1, on conditional syllogisms), we can draw a conclusion from either affirming the antecedent:

If you are from Brussels, you are a Belgian
You are from Brussels
∴ *You are a Belgian*

or by denying the consequent:

If you are from Brussels, you are a Belgian
You are not a Belgian
∴ *You are not from Brussels*

These are both valid syllogisms. They are valid because, fundamentally, a conditional proposition can be read as a simple one; our major premise in these two syllogisms could be written as, *All who are from Brussels are Belgians*, without any conditional at all. We then draw conclusions in the normal way.

But if we do the opposite, we can draw no valid conclusions. For example, if we say:

If you are from Brussels, you are a Belgian
You are a Belgian
∴ *You are from Brussels*

we commit the logical fallacy of *affirming the consequent*. Our conclusion may be perfectly true; but it does *not* follow from the premises. You may, for example, be from Bruges, or Antwerp.

Similarly, if we deny the antecedent:

If you are from Brussels, you are a Belgian

You are not from Brussels
∴ *You are not a Belgian*

we commit the fallacy of *denying the antecedent.* It may be perfectly true that you are not a Belgian; but it does *not* follow from the premises. You may not be from Brussels but still be a Belgian; you may just be from somewhere else in Belgium.

Subsection 5.5.6 Ad Hominem

This fallacy simply means that we attacked the person rather than the argument itself. It does not bear on the syllogism, as it's impossible to form even a pseudo-syllogism of this type; but it's certainly a common error in argument.

Jones says we shouldn't drink
But Jones is a drunk
∴ *Jones is wrong about drinking*

This doesn't even look like a syllogism, other than having three propositions; but once again, people commit this error all the time. Jones may well be a drunk; but that doesn't mean that he's wrong about drinking. The maker of the argument is totally irrelevant to the strengths of the argument itself.

Subsection 5.5.7 Ad populum

A common error in our democratic age, this fallacy argues that the more popular opinion is the right one. Yet it's clear that the number of people who agree with an opinion has no bearing on whether it is true. It typically runs something like this:

You are opposed to the theory of evolution
But nearly everyone agrees with the theory of evolution
∴ *You are wrong to oppose it*

You may very well be wrong to oppose it; but you're not wrong to oppose it because most people agree with it.

A similar fallacy is the *argumentum ad verecundiam*, or argument from shame. A Jew, for example, is considering becoming Catholic, and is discussing the matter with a rabbi. The rabbi tells him, "How can you oppose your ancestors? How can you go against the rabbis who wrote the Talmud? Maimonides, the great philosopher? How can you disrespect the memory of your grandfather, who died in a concentration camp?" Emotionally powerful, certainly; but not an argument against conversion to Catholicism. The fact that many respected and intelligent people chose to remain Jewish does not mean that remaining Jewish is the right choice.

Subsection 5.5.8 Post hoc ergo propter hoc

One of the most common of fallacies, *post hoc ergo propter hoc* translates literally to, "after this, therefore because of this". This fallacy is the source of a great deal of superstitution, for example. "I walked under a ladder, and then I nearly got hit by a car! It gave me bad luck!"

The mere fact that two things coincide in time does *not* mean that one caused the other. Coincidence is a real thing.

Chapter 6
Inductive Reasoning

Inductive reasoning has become one of modernity's greatest points of pride. Indeed, to many moderns the very notion of the syllogism is considered foolish, and inductive reasoning is the only type worthy of the name. The word *science*, once applicable to any systematic body of knowledge, now refers almost exclusively to those fields of endeavor which are built up by inductive reasoning.

As we discussed in Section 2.3, induction is the process of reasoning from particulars to universals. This can happen in three ways.

First, we might recognize a universal through particular examples. Geometry is an excellent case in point. When I am told that the sum of the interior angles of a triangle are equal to two right angles, I'm not immediately impressed by the fact. However, after several examples have been shown to me, I can intuit the fact, and when the proof has been done on multiple different triangles, I have no trouble with accepting it. But this is barely induction at all; it's really just a means of recognizing principles, and we need no further discussion of it.

Second, we can accumulate particular facts and use those to draw universal conclusions. This can happen in two primary ways. First, *complete induction*, by which we enumerate each and every particular and use that to make a conclusion about the universals. Second, *incomplete induction*, by which we do not enumerate each and every particular, but a large enough quantity to justify a conclusion about the universal. We will address each of these in turn.

Section 6.1
Complete Induction

When we perform complete induction, we are enumerating *all* of the possible particulars, and using that to draw a conclusion about the universal class that encompasses them.

complete induction

drawing a conclusion about universals from facts ascertained about every possible particular

As an example of complete induction, consider the beers brewed by a given brewery. Suppose there are three of them: an ale, a lager, and a porter. Each of them is very hoppy.

From these particulars, we conclude that *all of the beers of this brewery are hoppy*. The reasoning is thus:

> *This brewery's ale, lager, and porter are very hoppy*
> *The ale, lager, and porter are all the beers of this brewery*
> ∴ *All the beers of this brewery are very hoppy*

Aristotle, one of the earliest and most systematic of the logicians, defined this sort of reasoning as *proving the major term of the middle by means of the minor*; deduction, on the contrary, proves the major term of the minor by means of the middle.

There is usefulness in complete induction, as it makes formerly implicit knowledge explicit. On the other hand, it has weaknesses, as well. It does not establish any relation of cause and effect, and it establishes only a factual, not a necessary, relation. Our brewery, for example, could tomorrow produce a beer which is not very hoppy, and invalidate our conclusion.

It's also very possible that our complete induction is actually *in*complete, because we are missing one of the particulars and do not realize it. We saw, in Section 2.3, an example of induction involving metals and solidity at room temperature. For simplicity's sake, let's assume a premodern scientific situation which knows of only five metals: copper, iron, gold, silver, and tin. We form a syllogism in the attempt to make a complete induction:

> *Copper, iron, gold, silver, and tin are all solids*
> *Copper, iron, gold, silver, and tin are all metals*
> ∴ *All metals are solids*

Then somebody goes and discovers mercury; clearly a metal, and yet also clearly a liquid at room temperature. And our conclusion is false.

Induction, even complete induction, is thus less certain than deduction; for no matter how much data we have collected in an attempt to draw our conclusion, the possibility that some datum exists which will invalidate it will never be completely removed. We might be very, very certain; but absolute certainty is impossible.

Deduction, on the other hand, *can* produce absolute certainty. If we know that our premises are true and that our reasoning is valid, then our conclusion *must be true*, with an absolute certainty that induction cannot reach.

Modernity often melds the two, or fails to recognize any distinction at all. One example is the famous "God of the gaps" argument, which purports to prove the existence of God by the holes in our scientific understanding. The argument that modern philosophers both pose and shoot holes in goes like this:

> *Some process caused life*
> *Science cannot tell us that process*
> ∴ *Science cannot tell us what caused life*

Since science can't tell us what caused life, God is then posited as the explanation. The problem here is, of course, the minor premise; we're never completely certain of what science tells us. It cannot tell us *right now* what caused life; but further observation may very well change that, in which case the argument will fail.

Remarkably, modern scientists can always point out the failures in inductive reasoning of this type, but often remains strangely blind to the failures of inductive reasoning in other contexts. Defenders of scientism (by which I mean the notion that only inductive reasoning of the sort performed by the physical sciences can yield knowledge of the truth) can easily respond to many arguments in this way. But consider a more traditional (and deductive) proof of God's existence:

Contingent beings receive existence from another
That other must have existence in itself
∴ *Contingent beings receive existence from one which has existence in itself*

That being which has existence in itself (that is, essentially) we call God. (We're obviously glossing over a great deal of complexity here; but this does give us the idea.)

The standard inductive criticism simply doesn't work here. No amount of further observation can possibly disprove either of these premises; they're just *true*, independently of what we've seen or not seen. It's always possible that I've misstated a premise, or that I've drawn an invalid conclusion; but if the premises are true, and the syllogism is valid, then the conclusion *is true*. Induction cannot grant that sort of certainty.

Note also that induction may violate the normal rules of the syllogism and still be a valid induction. For example:

James I, James II, Charles I, and Charles II were headstrong
James I, James II, Charles I, and Charles II were all the Stuart kings
∴ *All the Stuart kings were headstrong*

A universal conclusion here is technically not warranted; the term *Stuart kings* is really *undistributed* in the minor premise, despite the use of the word "all". It's possible, however remotely, that a new Stuart will rise to the throne and falsify the minor premise. But it's still a valid *induction*, even though it fails as a *deduction*. It follows the principle of identity that, when two objects are identical with a third object, they are also identical with each other.

Violating the extension of the terms in the premises in the conclusion is not a problem for induction; after all, that's part of what induction is for (extending something we know about particulars into the universal). But we still need to show an identity; an undistributed middle, for example, is still a fallacy. In short, *we can violate syllogism Rule 2 in induction* and still have a valid conclusion, as long as we remember that this is *not* the same as a deductive syllogism.

None of this is to denigrate induction, which is an extremely useful means of coming to know reality. But we must keep in mind its weaknesses as well as its strengths.

Section 6.2
Incomplete Induction

By incomplete induction, we don't enumerate *all* the possible particulars to draw a conclusion about the class that contains them. Rather, we enumerate *some*, and use that to *extrapolate* to some universal statements.

> ***incomplete induction***
>
> drawing a conclusion about universals from facts ascertained about some of the possible particulars

It's worth noting here that the fewer the data points (that is, the fewer particular instances that are enumerated, when compared to the total number of actual instances), the weaker the induction is. If there are a thousand examples of a thing, and we enumerate nine hundred of them to make an incomplete induction, we have a pretty strong argument; but if we enumerate one hundred, we have a pretty weak one. The precise number of samples that makes the difference between *strong* and *weak* in this way will vary according to the field of study; it might take very few in, say, geology, but very, very many in biology. There is an entire branch of mathematics around *sample sizes* and such, called *statistics*. For now, however, it's important to keep this in mind and ask appropriate questions about *sample size* to judge the strength of the points being presented.

Aristotle's own example of this (also called *material induction*) remains pretty apt:

Pilots, charioteers, etc., who know their business are skillful
∴ *Generally, those who know their business are skillful*

The weakness with incomplete induction is even more evident than with complete induction: further observation may always bring about examples which disprove the conclusion. Furthermore, it has a further weakness: it's conclusion is only *general* or *probable*. It tells us that, if we grab from the pool of all those who know their business just one person, he will likely be skillful; but it's always possible that we'll get one who isn't. These conclusions are *useful*; but they do have that weakness.

Incomplete induction is very persuasive, however, and very easy to arrive at by observation, even though a rigorous deductive syllogism is more forceful and certain. Anyone can observe our argument above about pilots and charioteers and agree that this is likely correct; the same is *not* true about the sum of the interior angles of a triangle, which requires learning principles and analyzing deductions from them.

So what are the rules of incomplete induction? Essentially, the same as complete induction: the eight rules of syllogisms, except for Rule 2. Consider the following:

On Jan. 18 and on Jan. 24, we had rain
On Jan. 17 and on Jan. 23, the barometer dropped
∴ *On days after barometer drops, we have rain*

Yikes! Talk about insufficient data points! Certainly true; but let's ignore that for a moment and look at the logic. The dates, of course, are our middle term. Don't be fooled by the fact that they're different dates; we're just talking about different portions of the same time, "times around barometer drops and rainfall". But *days after barometer drops* is clearly *distributed* in the conclusion, while it's *undistributed* in the minor premise; the same is true of *we had rain* in the major. In deduction, this is an illicit process of the major (see Section 5.5.4); but in induction, this is perfectly fine. As long as our middle term is distributed at least once, we're still comparing identical things; we're just extending from particulars to universals.

Now, we might prove to be wrong, especially with only two examples from which we're universalizing. And while we might have posited a *correlation* between the drop in the barometer and the coming of rain, we haven't established (or even posited) that the one *causes* the other. But it's still a valid use of inductive principles.

How do we prove ourselves right or wrong about these particular cases? By *observation*, including that active type of observation known as *experimentation*.

Section 6.3
Observation and Experimentation

Experimentation is much touted by moderns, and with good reason; it's an excellent way to come to a knowledge of particulars, and to separate extraneous facts from essential ones. Ultimately, however, it's simply a method of controlled observation.

experimentation

observation by means of controlled tests

Rather than just watching things and seeing what happens, we set up a certain situation and see what happens. E.g., rather than just noticing things falling on their own, drop a few and see how fast they fall.

The scientific method makes use of these principles extensively; indeed, almost exclusively. This method consists roughly of five steps (though one sees it formulated in many different ways. (1) Question: pose the question in a properly narrow, clearly defined way. (2) Hypothesis: take an educated guess what the answer to the question may be. (3) Prediction: Make a prediction of what would be seen, if the hypothesis is correct. (4)

Experiment: do some controlled observation of what happens when the conditions of the prediction are made a reality. (5) Analysis: Analyze the results of the experiment to determine if the prediction was correct. Repeat, hopefully each time forming a hypothesis that proves closer to the results of the prediction.

However, this isn't a scientific textbook; we're not talking about "the scientific method" and its process for eliminating error. These are worthy topics; but they are worthy topics for another time, involving questions that go beyond our purposes here. Experimentation, and its particular form that is embodied in the scientific method, are all designed to work by taking advantage of the inductive principles which we will discuss in Sections 6.4.1, 6.4.2, 6.4.3, and 6.4.4. So let's move on to those.

Section 6.4
Methods of Making Inductions

Subsection 6.4.1 Method of Agreement

The *method of agreement* involves observing multiple instances of a phenomenon and determining what is common among them. If there is only one such circumstance, we can conclude that that circumstance is a cause, or an effect, of that phenomenon.

method of agreement

isolating a common circumstance among multiple instances of a given phenomenon, and concluding that common circumstance to be a cause or an effect of that phenomenon

Consider a group of people who are so astoundingly ignorant of the physical world that they cannot determine the cause of *melting*. They observe that multiple substances can be melted; iron will melt, ice will melt, copper will melt. They observe that all of these events consist of the same thing; that is, moving from the solid to the liquid state. In other words, they are multiple instances of the same phenomenon. So they isolate individual instances of all of these movements, and determine that only one thing is common to all of them: the application of a certain amount of heat. Ice needs the least heat to melt; copper, more; and iron, the most of all. But heat is required for all of them. The only thing *in agreement* is heat; so they conclude that heat is the cause.

This might be completely wrong, of course; it's possible that they will run into a substance which solidifies, rather than liquifies, when heat is applied to it. But they are very justified in their conclusion by applying the method of agreement.

Subsection 6.4.2 Method of Difference

The method of difference is the exact opposite process from the method of agreement. We take multiple instances which are identical in every way except one. If the phenomenon we are investigating occurs in one of those instances and not in the others, then the difference between those instances is a cause; an effect; or at least an indispensable part of that phenomenon.

method of difference

isolating a single differing circumstance among multiple instances, in one of which the phenomenon occurs and in the others of which it does not; and concluding that that single different circumstance is a cause; effect; or indispensable part of the phenomenon

Experimentation really shines when using this method. The famous experiment of Francesco Redi disproving spontaneous generation of flies from rotting meat is an excellent example. The theory was that rotting meat gave rise to flies, all on its own, because flies were observed to universally accompany rotting meat. Redi, however, doubted this; he thought it more likely that flies were attracted to rotting meat and laid their eggs in it, than that the rotting meat itself produced the flies. To prove this, he set up an experiment utilizing the method of difference.

He put some rotting meat in two jars, which were identical in every way but one: one of the jars was left open, giving flies access to it, and one was sealed off, so that flies would not have access to it. This was the only difference between the two jars. The open jar was soon swarming with maggots and flies, while the sealed jar was not. This single difference, then, explained the result: rotting meat didn't *produce* flies, it merely *attracted* them.

The experiment does not, of course, prove that Redi's theory about the source of the flies was right, necessarily. It might be that, to generate flies, rotting meat needs more air than was accessible in the sealed jar, or that the flies themselves suffocated before they could be fully generated and observed in the sealed jar. But it did prove one thing: being open to the air was necessary for meat to produce flies. That was the one difference, and that explained the result.

Subsection 6.4.3 Method of Concomitant Variations

This next method is a bit more complex. When one factor varies in some particular way as another factor varies in a particular way, those phenomena are connected in some way, whether as a cause, an effect, or some relation.

method of concomitant variation

observing two factors which vary in some way related to one another, and concluding that they are related as cause, effect, or in some other relation

Sir Isaac Newton's law of gravitation may be the most obvious example. Observing the force between two massive objects, Newton noted that the farther away the two were, the weaker the force between them; specifically, the force varied inversely with the square of the distance between the objects. In other words, the farther away the two objects were, the weaker the force. From this he concluded that gravitational force is, in some manner, *caused by* or *related to* the distance between two objects.

This doesn't mean, of course, that closeness definitely generates gravitational force. But it does confirm a relation between the two.

Subsection 6.4.4 Method of Residues

The last inductive method we'll examine is the method of residue. As its name implies, it works by taking away from a phenomenon what we know to be the effect of other things, and concluding that whatever is left over is the effect of the remaining causes.

method of residues

taking away what we know *not* to be causing the phenomenon, and concluding that what is left over *is* causing the phenomenon

This is the weakest of inductive methods, because we can never really be sure that we're taking away *everything* that isn't related. It also may not be possible; certain things result in multiple effects, and so taking it away may weaken or eliminate the phenomenon that we're trying to isolate. But it can certainly help to increase our certainty in whatever conclusion we're eventually able to draw, or have drawn, from other methods.

Section 6.5 Example and Analogy

Closely akin to induction, and really subordinate to it, are *examples* and *analogies*, by which we draw conclusions that are not certain, but which nevertheless suggest certain things to be true. These are *weak* forms of induction, and are primarily useful in *argumentation*, or *rhetoric*, which as a whole is a subject for a different day. But they are ways in which we make arguments about things, and therefore they will be briefly examined here.

Subsection 6.5.1 Example

Example is very much akin to incomplete induction, when we draw a conclusion about a universal from a few, or even one, particular instance.

example

drawing a general conclusion from a particular example

Consider the following:

Mr. Smith is a good man
Mr. Smith is a Congressman
∴ *Congressmen are good men*

What a ridiculous conclusion! Deductively, this syllogism is clearly fallacious, by reason of an illicit process of the of the minor (*Congressmen*). As an incomplete induction, it's a poor one; one example is quite insufficient to draw a conclusion about even the general character of the more than five hundred Congressmen in the United States. But, despite its absurdity, it's a conclusion that we frequently draw. It's a conclusion by example.

The more examples from a given class we have, of course, the more reliable this type of conclusion will be (though it will never be completely reliable, even when our number of examples equals the number of instances in the class, because a new member may always be introduced which will invalidate it). This is true of incomplete induction, as well, of which *example* is merely a special case. It's often referred to as *Socratic induction*, because Socrates himself loved to engage in it.

Overextending examples is rampant. What Catholic has not heard perfect instances of it from acquaintances? "I used to be Catholic, but my priest as a child cared about nothing but money, and he turned out to be a child molester, too." Or, "I'm not superstitious. All I know is that when I've used this pencil I've passed my tests, and when I used a different one last week I failed; so I'm going to use this pencil from now on." It's likely the most abundant source of bad reasoning in the world.

Yet it's not, for that reason, useless. It can be a very useful shorthand, *provided that we remember its limits*. Examples are *examples*; we will nearly always encounter counterexamples which prove that they are not general. Remember that, and we can use examples as needed, appropriately, without fooling ourselves into thinking we've learned more than we have.

Subsection 6.5.2 Analogy

Analogy is so closely akin to example that it's not always easy to distinguish them. *Example* extends from one particular instance to a general assertion; *analogy* extends from one particular instance to another particular instance of a different type.

analogy

from a particular instance, drawing a conclusion about a different particular instance

Consider the following:

Mr. Smith is to Congress as Mr. Jones is to Parliament
Mr. Smith is bound to vote in Congress
∴ *Mr. Jones is bound to vote in Parliament*

Example is prone to mislead; analogy is even more so. We're concluding not from common membership in a given class, as we do with example; we're concluding from a common, similar relationship, which is farther removed.

That said, like example, analogy is sometimes very helpful and useful, even if it's not strictly demonstrative. Take a very commonly-noted example:

The contrast between Judas Iscariot's belief and his practice did not make his belief in Christ false
But Judas is to Christ just as the bishops are to Christ
∴ *The contrast between a bishop's belief and his practice do not make his belief false*

If we accept that Judas's relation to Christ is the same as the bishop's relation to Christ, at least in this particular, than the conclusion certainly validly follows. But analogy is one of the weakest forms of argument *objectively*, even though it often carries great psychological force.

Section 6.6 Authority

One additional type of inductive reasoning is the *argument from authority*, by which we agree with a proposition based partly or wholly on who is presenting the proposition to us.

argument from authority

the position that a proposition is true based on the proposer of it

In a certain sense, this argument is akin to the *argumentum ad hominem* (see Section 5.5.6), in that the reliability of an argument is based upon the identity of who proposed it. However, fundamentally, the argument from authority is quite different.

By an argument *ad hominem*, we argue that the proposer of a position is bad, and therefore his position is bad; by the argument from authority, we argue that we should accept a proposition *because he who proposes it is in a position to know the truth about it*. These are two entirely different things. By an argument from authority, we say something like the following:

The doctor tells me that smoking is unhealthy
The doctor is very knowledgeable about health
∴ *Smoking is unhealthy*

This is a perfectly reasonable way to come to a conclusion, particularly in those cases where the knowledge necessary to come to a conclusion on one's own is voluminous or difficult to acquire. Your author, for example, has done none of the experimentation necessary to establish the value of the gravitational constant, or the number of chromosomes in a human cell; I accept that the former is 6.67408×10^{-11}, and that the latter is forty-six, on the authority of those who *have* done them.

It is quite fashionable to object to arguments from authority as invalid; but this is quite wrong. The argument from authority is weak in comparison to other types of argument; however, it is certainly not powerless. It is a valid argument that should be addressed on its merits.

Those merits are *the basis for the authority in question*. (Of course, one can always argue from the matter itself, independently of the authority, as well.) So if I make the following argument:

My mother says that coffee is bad for me
My mother knows a lot about health
∴ *Coffee is bad for me*

a perfectly valid response would be, "But your mother knows very little about health." Also, "But your mother's knowledge of health is based on her nursing school forty years ago, and subsequent research has changed medicine's opinion about coffee." An *invalid* response would be, "That's just an argument from authority." Of course it is; but if the authority is good, so is the argument.

Even very good authority can be wrong, of course. A famous example is a very simple one: the number of chromosomes in the human cell. For dozens of years, scientists very confidently asserted, and printed in textbooks and scientific papers, that humans had precisely four dozen chromosomes in their cells. It was discovered later on, however, that the correct number was two fewer, forty-six.[1] Prior to this new, more accurate count, claiming that the number was forty-eight on the basis of authority would have been perfectly valid, and quite certain; it would nevertheless have been wrong.

But the mere fact that authority is not *infallible* does not mean that it's *invalid*. When an argument from authority is made, the proper response is independent reasoning or analysis of the strength of the authority, not mere dismissal of the argument.

Section 6.7
Dialectic

Dialectic is a form of argumentation which found its earliest (and, arguably, its best) formulation in the Socratic dialogues, and consists of the advancement and refutation of arguments, counterarguments, and so forth until one can reach a final conclusion:

> ***dialectic***
>
> the presentation and refutation of arguments and counterarguments regarding a given topic

The method of dialectic is often equated with "the Socratic method".

Dialectic is not really part of induction, nor is it part of deduction, as it makes use of both in its pursuit of the truth. The best example for the Catholic Christian is the work of St. Thomas Aquinas, which is nothing less than a repeated dialectic.

- A question (*quæstio*) is posed, whereby St. Thomas asks whether (*vtrum*) something is true.
- A few arguments follow with a provisional answer to the question (which are called *objectiones*, "objections", because they will eventually be proven wrong).
- St. Thomas will then present the correct argument, often an argument from authority, in very brief form (*sed contra*).
- An extended argument in favor of the correct argument follows (*Respondeo quod*).
- Each of the provisional arguments will be answered (*ad primam*, etc.).

This written form is based on the oral *disputationes* (arguments) that were conducted at medieval universities all the time. This is a *dialectic*, in which we come to a knowledge of the truth by proposing various arguments, shooting them down, bringing up responses

[1] See, e.g., Stanley M. Gartler, *The chromosome number in humans: a brief history* in 7 Nature Reviews 655 (2006).

to the attempts to shoot them down, and so forth until we have arrived at arguments that carry real force.

Few of us, of course, will have the mastery in this form of argument possessed by Socrates or the medieval masters. But we engage in this sort of thing all the time. Our internal monologues are full of it; the pro-con lists that many of us make regarding our practical choices are examples of it. Dialectic is central to our coming to the truth.

We can use dialectic with syllogisms and formal inductions; or we can use dialectic to come up with syllogisms and formal inductions; or both. It's useful not only for establishing our arguments, but even for determining what direction we think a discussion of the matter should go. It's also immensely useful for convincing others about our arguments; but that is a matter for another study, rhetoric, which is beyond our scope here.

Chapter 7
Conclusion

Here we have examined all the principle parts of logic, and even dipped our toes into the world of rhetoric, though obviously much more remains to be said about that topic than we have reviewed here. We can spot an argument; examine its premises; and determine whether it is valid or invalid as a demonstration. We can determine whether propositions are necessary or merely probable, and if the latter how strong they are, and thus gauge the strength of a valid argument, and therefore how it should convince us. All told, we can think *rigorously* and *carefully*, and thus come to good conclusions, if there are good conclusions to be had.

This last point is one which often pulls a lot of scorn. *Rigorously*? *Carefully*? Shouldn't we put the "human" element, the "heart", into our reasoning? A strong stream of fiction, exemplified in the dynamic between James Kirk and Spock, even asserts that it's the *ir*rational part of our natures, rather than the rational part, that makes us truly human. We are asked to "think" with our emotions, rather than to reason and judge with our intellects. That, we are told, is true humanity.

But the Catholic Church, and the Western civilizational tradition of which it is the apex, could not disagree more strongly. Our passions are at the *service* of our reason; they are not its master. It is precisely our *reason* which makes us truly human; and our ability to correctly exercise our reason is the quintessential ability of our humanity. Everything that is good for us—including the applications of our emotions to particular situations—comes from the exercise of our *reason* and our *will*.

It is impossible, then, to overestimate the importance of logic, the study of the rules of right reasoning, to intellectual life. And as we mentioned in the introduction, the rules of logic are neither many nor difficult; a brief study suffices for the basics of them, and a thorough knowledge of those basics pays huge dividends in *all* pursuits of human knowledge.

It is our prayer that this little book has served that purpose. May St. Thomas Aquinas, the Angelic Doctor, patron of intellectual pursuits, pray for all of us who use it and who study from it.

Chapter 8
Answers to the Exercises

Exercises 3-1

1. This is almost comically simple; the term *refuse* is clearly being used *equivocally*. **2.** The first phrase refers to a literally ball; the second refers to the initiative for further action. The terms are being used *analogically*. **3.** Univocally **4.** Don't be fooled here; *blind* is being used analogically to actual, physical blindness; but the common term, *love*, is being used univocally; in both cases we are referring to *love* meaning "unconcerned with superficial appearance". *Univocally*. **5.** Clearly, the love we are referring to in these two sentences is related but not the same (hopefully). *Analogically*. **6.** Likely, *univocally*.

Exercises 3-2

1. The options here, of course, are vast. Think *four-legged*, *furry*, *placental*, *carnivorous*, *mammal*, *four-toed*, *warm-blooded*, *clawed*, *domesticated*, and any others you might come up with. **2.** Fill this in as appropriate. **3.** For species, perhaps *snake*, *turtle*, *lizard*, *alligator*. For genera, perhaps *cold-blooded*, *egg-laying*, *terrestrial*, *scaled*. **4.** They share *substance*, *living*, *sensate*, *legged*, *clawed*, *vertebrate*, *oviparous* ("egg-laying"). They do not share *warm-blooded* (Herbie is cold-blooded); *feathered* (Herbie is scaled, not feathered); *flying* (Herbie can swim, but he can't fly).

Exercises 3-3

1. Considered as *oviparous* (egg-laying), turtles and parrots differ in significant ways. Both lay their eggs in dry places (that is, unlike fish and amphibians, which lay their eggs in water). Bird-eggs have hard, brittle shells, while turtle eggs are leathery. So considered as *oviparous*, *turtle* could be defined as *leathery egg-layers*, while *parrot* could be defined as *hard-shelled egg-layers*. **2.** Considered as members of the genus *this woman's children*, the specific difference of her son is *male*, and of her daughter is *female*. **3.** The genus *socket-wrench* has two species, *metric* (specific difference "measured in metric units") and *standard* (specific difference "measured in standard U.S. customary units"). We can consider each of these species as genera. The genus *standard socket-wrenches* has several species—quarter-inch, eighth-inch, and so forth. The genus *metric socket-wrenches* has several species, as well—twelve millimeter, fifteen millimeter, and so forth. The specific difference of each of these species, in both genera, is the actual measurement. **4.** Genus *writing implements*; species *pen*, *pencil*. Genus *writing surfaces*; species *legal pad*, *notebook*, *post-it*. Genus *computing devices*; species *calculator*, *computer*, *laptop*, *smartphone*.

Exercises 3-4

1. Choose a genus and a specific difference. For example, *canine* as the genus (to include coyotes, wolves, foxes, etc.), and *domesticated* as the specific difference. **2.** reptile with a strong shell **3.** primate with a long tail **4.** literature organized by lines **5.** A tougher nut to crack. Its genus might be *calculating device*, and its specific difference *electronic*; this would make sense if we have some non-electronic calculating devices, like an abacus and a slide rule. Its genus might instead be *electronic device* and its specific difference be *for arithmetic*, if we have many electronic devices besides the calculator. **6.** The genus (*animal*) is fine, as far as it goes; but the specific difference (*scaled*) is too broad; it does not express what makes a turtle different from all other animals. It applies to many other animals, such as fish, lizards, alligators, crocodiles, and so forth. So the definition is *inadequate*. **7.** The genus, *tool*, seems appropriate. The specific difference, *for pounding nails*, also seems appropriate; that's what makes it a hammer and not some other tool. So this definition is *adequate*. **8.** The genus, *tool*, is appropriate. However the specific difference, *for fastening*, is too broad; it does not tell us what makes screwdrivers different from all other tools. A *hammer*, potentially *wrenches*, and other tools are equally tools for fastening. So this definition is *inadequate*. **9.** A famous example, drawn from a debate between Plato and Dioegenes. Supposedly, after Plato proposed this as a definition for *man*, Diogenes plucked a chicken, brought it to Plato's school, and threw it over the wall, crying, "Behold, Plato's man!" Which story does highlight the problem with this definition. The genus, *biped*, is poorly chosen; it groups men with many very different creatures, such as birds and some lizards, to which we are clearly not very similar. Further, the specific difference, *featherless*, is badly chosen; it fails to identify what makes *man* different from all other bipeds. For example, bipedal lizards are also featherless. This definition, then, is *inadequate*.

Exercises 3-5

1. *Accidental*; hair color is no part of what it means to be a *man*, and also exists in many other creatures. **2.** *Proper*; all turtles have shells, and other animals do not have turtle-like shells. A creature without a shell would not be a turtle. Indeed, the shell might not only be *proper*, but *essential*. **3.** *Accidental*; some hammers have claws, some have peens, some have nothing, but all are equally hammers. **4.** *Accidental*; a knife is a tool used for cutting, and it may not have a point but still be a knife. Both butcher knives (no points) and daggers (points) are knives. **5.** *Accidental*; apples may be red, pink, yellow, or green, and still equally be apples. **6.** *Proper*. Man by nature desires to know, and no other creature desires to know (in the same intellectual sense) as the rational animal man. May even be *essential*; but arguably the *desire to know* is not part of our nature, but merely the capacity to do so.

Exercises 3-6

1. *Relation* **2.** *Substance* **3.** *Quality*; color arises from the *form* of the thing. **4.** *Habit*; it's not part of the thing itself, but is around the thing and so close to it as to be part of it and to characterize it. **5.** *Quantity* **6.** *Relation*, insofar as we're asserting that it's *my* dog; and *substance*, insofar as we're asserting that it's my *dog*. **7.** *Timewhen* **8.** *Habit* **9.** *Placewhere* **↊.** *Substance* **↋.** *Position*; we're referring specifically to the arrangement of its parts. Not *action*; though we're using an active verb here, the subject isn't actually doing anything. We're just talking about how his parts are currently arranged. **10.** *Action*, insofar as we're referring to the throwing by the subject; *passion*, insofar as we're referring to the *being thrown* of the ball. **11.** *Quality* **12.** *Passion*; an action was done upon him

Exercises 4-1

1. *Contingent* **2.** *Necessary* **3.** *Necessary* **4.** *Contingent* **5.** Trickier than the last few. All mammals are, in fact, warm-blooded. However, that's not *what mammals are*, which is rather animals with mammary glands for the nursing of their young. While *in fact* there are no mammals which are cold-blooded, given this definition there is no reason that there could not be. So the answer is *contingent*; warm-bloodedness is proper to mammals, but not essential. **6.** *Necessary*; being rational is part of what a human being is.

Exercises 4-2

1. *Universal*; the proposition indicates that *all* dogs are mammals. **2.** *Universal*; all propositions with a singular, particular subject are universal. All *Rover* eats dog food; this is true because there is only one Rover. **3.** *Particular*; indeed, the insertion of the modifier "some", "many", or any similar adjective will often make the determination between particular and universal very easy. **4.** As expressed, *universal*; however, the proposer almost certainly does not mean to say that *all* malodorous things are literally rotten, so the meaning is likely *particular*. **5.** Almost certainly *universal*, in both expression and intent. **6.** *Universal*, most probably in expression and intent, but certainly in expression. Possibly the speaker refers only to certain lives, in which case, *particular*. **7.** In expression, *universal*; however, the speaker does not mean to say that every necessity will bear inventions, so *particular* in intent.

Exercises 4-3

1. *Affirmative* **2.** *Negative* **3.** *Negative* **4.** *Negative* **5.** At the risk of being ridiculous. This statement is equivalent to *all turtles are reptiles*, so it is *affirmative*.

Exercises 4-4

1. Particular affirmative; I. **2.** Universal affirmative; A. **3.** Universal negative; E. **4.** Tricky! The proposition essentially means "All men are dogs". Universal affirmative; A. **5.**

Particular negative; O. **6.** Universal negative; E. **7.** Universal negative; E. **8.** Particular negative; O. **9.** Particular affirmative; I. **↊.** Particular affirmative; I. **↋.** Tricky! A *negative universal* subject of this type ("no computers") should translate to an affirmative universal subject ("all computers") and a negative verb ("are not"). So the proposition really means, "All computers are not horses". Universal negative; E. **10.** Universal affirmative; A. **11.** Universal affirmative; A. **12.** Particular affirmative; I. **13.** Universal negative; E. **14.** Universal negative; E. **15.** Particular affirmative; I.

Exercises 4-5

1. *Not convertible*; this is a particular negative proposition. **2.** *Some animals are men* **3.** *No women are men* **4.** *Not convertible*; this is a particular negative proposition. **5.** *Some tools are hammers*

Exercises 4-6

1. This is an A and an E; they are *contraries*. **2.** An A and an O; they are *contradictories*. **3.** An E and an O; these are *subalterns*. **4.** An A and an O; they are *contradictories*. **5.** An O and an I; these are *subcontraries*. **6.** An A and an I; these are *subalterns*. **7.** An O and an E; these are *subalterns*. **8.** An O and an A; these are *contradictories*. **9.** This is an A. We can conclude the falsity of the contradictory O ("some men are not apes"); we can conclude that its contrary E ("no men are apes") is false; and since it is universal, we can conclude that its subaltern I ("some men are apes") is also true. **↊.** Translating this to "All men are not reptiles", we see that it is an E (universal negative). Thus we can conclude that its I contradictory ("some men are reptiles") is false; we can conclude that its O subaltern ("some men are not reptiles") is true; and that its contrary A ("All men are reptiles") is false. **↋.** This is an I, particular affirmative. We can conclude that its contradictory E ("all men are not reptiles") is true, and that its subcontrary O ("some men are not reptiles") is also true. Because it is particular, we can conclude nothing about its subaltern, and since it's particular, it has no contrary.

Exercises 5-1

1. *Valid.* **2.** *Invalid*; Rule 3 tells us that the middle term cannot appear in the conclusion, but here the middle term, *oratory*, does appear in the conclusion. **3.** *Invalid.* (Also, *false*; but for our purposes that's beside the point.) Rule 4 tells us that the middle term must be distributed in at least one of the premises. Here, it is undistributed in the major premise, because while some Jews undoubtedly do hate Christians, it would idiotic to say that *all* Jews hate Christians. It is also clearly undistributed in the minor premise, because it is the predicate of an affirmative proposition. This is the fallacy of the *undistributed middle*. **4.** *Valid.* Both premises and the conclusion are all ridiculous; but the syllogism is valid nevertheless. **5.** As always, don't be deceived by the obviously true conclusion! Shoemakers are not gophers; but that doesn't follow from these premises. From two negative premises, no conclusion can be drawn; this syllogism violates Rule 5. *Invalid.* **6.** *Invalid*; Rule 1 states that a syllogism can have three and only three terms. This has four, because *pages* is used equivocally, meaning one thing in the major premise and another thing in the minor. This is the fallacy of *equivocation.* **7.** *Invalid.* Rule 7 tells us that we can draw no conclusion from two particular premises. Rule 4 also tells us that the middle term must be distributed at least once. This syllogism violates both. While the conclusion is undoubtedly true, our premises leave the possibility that the group of cabbies who are rude and the group of cabbies that are men are entirely different, and thus we cannot draw a conclusion. **8.** *Valid.* **9.** *Invalid.* Rule 6 tells us that we cannot draw a negative conclusion from affirmative premises. We *can* conclude from these premises that "some fruits are sour"; but it might be that *all* fruits are sour, so a negative conclusion cannot be drawn. **↊.** *Invalid.* Rule 2 tells us that no term in the conclusion can have a greater extension than it has in the premises; but here *fierce* is distributed in the conclusion (since it's the predicate of a negative proposition), but it is undistributed in the major premise (since it's the predicate of an affirmative proposition). This is an *illicit process of the major*, or simply *illicit major*.

Exercises 5-2

1. First figure; AAA; *valid.* **2.** Third figure; AEE; *invalid.* **3.** First figure; AAA; *invalid.* **4.** Second figure; EAE; *valid.* **5.** First figure; IAA; *invalid.* **6.** Second figure; AEE; *valid.* **7.** First figure (take care and note that the minor premise is actually listed *before* the major here); AAA; *valid.* **8.** Third figure; AAI; *valid.* **9.** First figure (again, take care and note that the minor premise is here *before* the major); EEE; *invalid.* **↊.** First figure; AAA; *invalid.* **↋.** Second figure; IEO; *invalid.* **10.** First figure; III; *invalid.* **11.** Third figure; EAE; *invalid.* **12.** First figure; AAA; *valid.* **13.** First figure; AIO; *invalid.* **14.** Second figure; AII; *invalid.* **15.** Third figure; EAO; *valid.* It helps clarify the matter to rephrase "no mosquitos are pleasant" to "all mosquitos are not pleasant". **16.** First figure; AEE; *invalid.* **17.** Fourth figure; AAI; *valid*, even though false.

Exercises 5-3

1. *Valid*; it affirms the antecedent and therefore the consequent. **2.** *Invalid*; it's denying the antecedent. We know that beasts with fur are mammals; but not that *only* beasts with fur are mammals. Therefore, we cannot conclude whether the whale is a mammal by noting it does not have fur. **3.** *Valid*; we affirm the antecedent, and therefore the consequent. **4.** *Invalid*; we're denying the antecedent. The plague isn't the *only* thing that puts us in danger of death. **5.** *Valid*; we deny the consequent, enabling us to deny the antecedent. **6.** *Invalid*; affirming the consequent. Reptiles have scales, but we don't know that *only* reptiles have scales.

Exercises 5-4

1. *Valid*, assuming that the major premise accurately describes these two as the only alternatives. **2.** *Invalid*. There are *three* alternatives in the major premise, not two; denying one means that one of the other two must be true, but not which. The correct conclusion is "I am either the same age or younger". **3.** *Invalid*; the major premise does not pose three *exclusive* categories, because Massachusetts is part of New England. So affirming that he lives in New England does not exclude him living in Massachusetts. **4.** *Valid*. **5.** *Invalid*. This is a dilemma. However, it doesn't work because the two disjuncts in the major premise are not exclusive. It is, of course, possible to both pray and work. **6.** *Valid*

Exercises 5-5

1. The term *must do as Christ did* is *undistributed* in the conclusion of the first syllogism, which means that it must be taken as *undistributed* in the major premise of the second. That means that *doing as Christ did* is an undistributed middle in the second syllogism, and the conclusion is invalid. It's certainly true that Christians must help the poor; but this logical argument doesn't prove it.

Colophon

This document is set in EB Garamond 7/10, with decorative initials in Linux Libertine. It was designed and produced using a variety of interlocked traditional Unix scripts, and imprinted using the LaTeX document preparation system, specifically the LuaLaTeX form.